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THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

LXXI

JANUARY 1902

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On the Dielectric Constant of Dilute Electrolytic Solutions — A. de FOREST PALMER — 38

A Study of the Action of Sound Waves on Unignited Jets of Gas — P. L. TUFTS — 57

Some Diffraction Photographs — W. S. FRANKLIN — 61

Note: *Rudolf Koenig* — 62

New Books: *Schuster*: Advanced Exercises in Practical Physics — 64

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COTENTS OF VOLUME XIV

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LXXII. FEBRUARY, 1902

The Velocity of Ions drawn from the Electric Arc. II. C. D. CHILD . . 65

The Density and Surface Tension of Liquid Air. CHAS. T. KNIPP 75

Crystallization under Electrostatic Stress. PAUL R. HEYL 83

Some Experiments on the Behavior of Dielectrics when Subjected to High Potentials. J. S. SHEARER 89

A Carbon Electrolytic Interrupter. A. H. TAYLOR 118

New Books: *Travers:* Experimental Study of Gases; *Jaumann:* Elektrizität und Licht; *Herdman* and *Nagler:* Manual of Electrotherapeutics; *Tory* and *Pitcher:* Laboratory Physics; Schweizerische Bergbahnen 122

LXXIII. MARCH, 1902

The Elastic Properties of Helical Springs. J. W. MILLER, Jr. 129

On the Boundary Conditions of the Electrical Field. WM. DUANE 149

Note on Kirchoff's Theory of Magnetostriction. S. SANO 158

The Vowel A^{E} (as in Hat). LOUIS BEVIER, JR. 171

Magnetization of Steel at Liquid Air Temperatures. C. C. TROWBRIDGE . . 181

New Books: *Jones:* Outlines of Electrochemistry 192

LXXIV. APRIL, 1902

The Specific Heat of Solutions. III. A Form of the Pfaundler Calorimeter. W. F. MAGIE 193

On Some Optical Properties of Asphalt. E. L. NICHOLS 204

The Vowel E. LOUIS BEVIER, JR. 214

The Velocity of the Ions from Hot Platinum Wires. C. D. CHILD 221

A New Form of Cavendish Balance. G. K. BURGESS 247

LXXV. MAY–JUNE, 1902

The Value of the Gravitation Constant. G. K. BURGESS 257

The Velocity of the Ions from Hot Platinum. II. C. D. CHILD 265

Contributions to the Study of the Induction Coil. J. E. IVES 280

Surface Color. R. W. WOOD 315

Index 319

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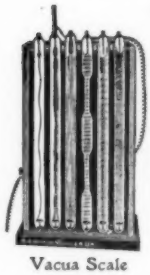

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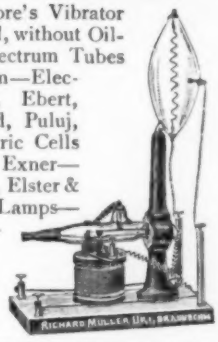

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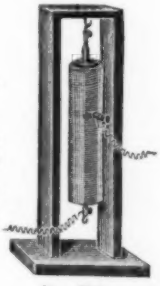

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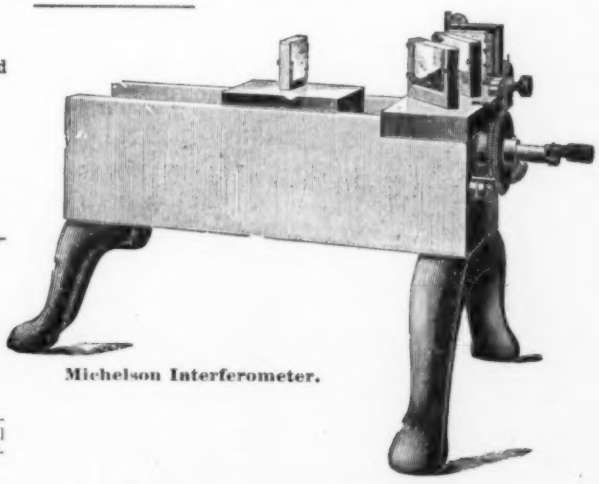

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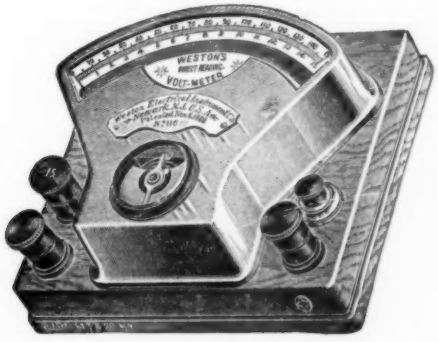

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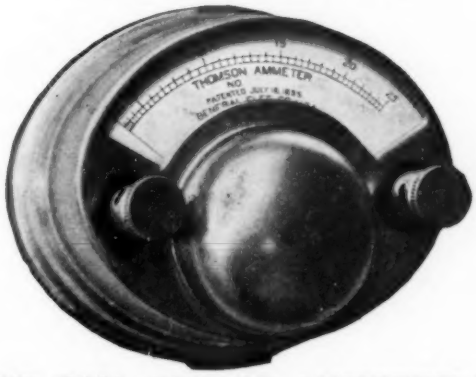

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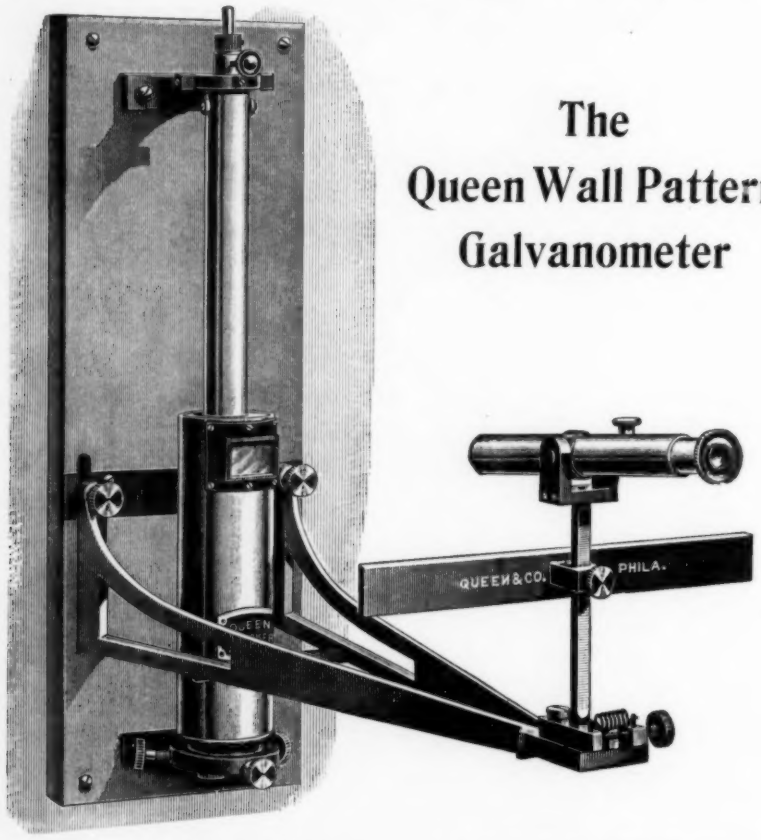

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THE

PHYSICAL REVIEW.

ON THE RELATION BETWEEN DENSITY AND INDEX OF REFRACTION OF AIR.

By Henry G. Gale.

IF a body is compressed or its temperature varied, its density, d, will change and so also will its index of refraction, n. The problem of finding the relation between the quantities d and n is an old one and one attended by some difficulties. In the case of solids and liquids the density can be made to vary only slightly and experiments cannot be conducted over wide limits. In the case of gases the density can be given a wide range, but the index of refraction differs but little from unity, and its variation with changes of density is small.

Gladstone and Dale[1] first proposed the very simple relation,

$$\frac{n-1}{d} = \text{const.} \tag{1}$$

This is the relation which would be expected if transparent substances owe their refractive power to their molecules alone, and if the molecular index of refraction does not change with the density.

Jamin[2] proposed the formula,

$$\frac{n^2-1}{d} = \text{const.} \tag{2}$$

a relation which follows necessarily on the basis of the emission

[1] Gladstone and Dale, Phil. Trans., p. 887, 1858; Phil. Trans., p. 317, 1863.

[2] Jamin, Ann. de Chim. et de Phys., 3e série, T. LII., p. 163, 1858; T. LXI., p. 358, 1861.

theory. For gases this relation is almost the same as that of Gladstone and Dale, since

$$\frac{n^2-1}{d}=\frac{(n-1)(n+1)}{d}=\frac{2(n-1)}{d}$$

$n+1$ being very little different from 2.

Lorentz,[1] on the basis of the electro-magnetic theory of light, has proposed the following formula,

$$\frac{n^2-1}{(n^2+2)d}=\text{const.} \tag{3}$$

which comes at once on substituting n for D in Clausius' equation $\frac{D+2}{D-1}d=\text{const.}$, where D is the dielectric constant. It is seen that the relation (3) amounts to the same thing as relation (1) in the case of gases since

$$\frac{n^2-1}{(n^2+2)d}=\frac{(n-1)(n+1)}{(n^2+2)d}=\frac{2}{3}\frac{(n-1)}{d}$$

because $n+1$ is very nearly equal to 2, and n^2 differs but very little from unity.

Jamin[2] was among the first to conduct experiments designed to test these relations. He measured the index of refraction of water at different pressures and found $\frac{n^2-1}{d}$ very nearly constant, but Mascart[2] showed that Jamin's results satisfied still better the equation of Gladstone and Dale.

Quincke[3] verified the results of Mascart for water and found that for glycerine, Rüböl, carbon disulphide, alcohol and ether, equation (2) always gave too large computed values and equation (3) too small values, while the values computed from equation (1) were nearly correct, being sometines a little too large and sometimes too small.

[1] Lorentz, Wied. Ann., Bd. IX., S. 641.
[2] Mascart, Comptes Rendus, LXXVIII., p. 801; Pogg. Ann., Bd. CLIII., S. 154.
[3] Quincke, Wied. Ann., Bd. XIX., S. 401; Bd. XLIV., S. 774.

Röntgen and Zehnder[1] obtained results in agreement with Quincke's for water, but they found that for the other substances equation (1) gave results consistently a little too large, but nearer than equations (2) and (3).

Quincke attempted to measure the change in the index of refraction of solids with a change of pressure, but the variations he was able to produce were too small to be measured accurately.

Rudberg,[2] Fizeau,[3] Stefan,[4] F. Vogel,[5] Pulfrich,[6] Offret[7] and others have investigated the change in the index of refraction of many crystals and transparent bodies as the temperature was varied over wide limits.

Jamin,[8] Damien,[9] Pulfrich,[10] Rühlmann,[11] Wüllner,[12] B. Walter,[13] Ketteler[14] and others have found that the maximum index of refraction of water occurs at a temperature somewhat below that of maximum density, and have investigated the relation of n and d for water from temperatures below 0° C. to 100° C. They found that the quotient $\frac{n-1}{d}$ diminished slightly as the temperature was increased. The observations of Ketteler and Pulfrich for water give, for sodium light, the following values:

	$\frac{n-1}{d}$	$\frac{n^2-1}{d}$	$\frac{n^2-1}{(n^2+2)}\cdot\frac{1}{d}$
At — 10°	.33447	.78061	.20656
At 10°	.33363	.77922	.20618
At 56.8°	.33274	.77464	.20585
At 95.2°	.33189	.76998	.20583

[1] Zehnder, Wied. Ann., Bd. XXXIV., S. 91; Röntgen und Zehnder, Wied. Ann., Bd. XLIV., S. 1 und S. 24.

[2] Rudberg, Pogg. Ann., Bd., XXVI., S. 291.

[3] Fizeau, Ann. de Chim. et de Phys., 3e série, t. LXVI., p. 429; 4e série, t. II., p. 143; Pogg. Ann., Bd. CXIX., S. 87 u. 297.

[4] Stefan, Berichter der Wiener Akademie (2), LXIII., S. 223.

[5] F. Vogel, Wied. Ann., Bd. XXV., S. 87.

[6] Pulfrich, Wied. Ann., Bd. XLV., S. 609.

[7] Offret, Bulletin de la soc. franç. de min., t. XIII., p. 405, 1890.

[8] Jamin, Comptes Rendus, t. XLIII., p. 1191; Pogg. Ann., Bd. C., S. 478, 1857.

[9] Damien, Ann. de l'école norm., sup., 2e série, t. X., p. 257, 1881.

[10] Pulfrich, Wied. Ann., Bd. XXXIV., S. 326, 1888.

[11] Rühlmann, Pogg. Ann., Bd. CXXXII., S. 1 u. 177.

[12] Wüllner, Pogg. Ann., Bd. CXXXIII., S. 1.

[13] B. Walter, Wied. Ann., Bd. XLVI., S. 423.

[14] Ketteler, Wied. Ann., Bd. XXXIII., S. 353 u. 506.

When instead of n the constant A of Cauchy's formula, $n = A + \frac{B}{\lambda^2}$ was used, the following relations were obtained:

	$\frac{A-1}{d}$	$\frac{A^2-1}{d}$	$\frac{A^2-1}{A^2+2}\cdot\frac{1}{d}$
At 10°	.32516	.75867	.20185
At 20°	.32463	.75717	.20159
At 30°	.32445	.75644	.20154

Wüllner found that the best agreement was given by the formula $\frac{A-1}{d} = a + bt$, the constant b being negative for glycerine, alcohol and carbon disulphide, but positive for a saturated solution of zinc chloride in water. Knops,[1] Weegmann[2] and Dufet[3] obtained similar results.

Ketteler[4] proposed the formula $\frac{n^2-1}{n^2+x}\cdot\frac{1}{d} = \text{const.}$, and computed x from the observations of Knops and Weegmann who had extended their investigations to a great number of organic substances. Ketteler found values of x ranging from 2 to 8.4.

On varying the temperature of air, Mascart[5] found that the index of refraction decreased more rapidly than the density with increasing temperature, but von Lang[6] obtained an opposite result by measuring the index of refraction directly from air at a high temperature to air at ordinary temperatures.

Benoit[7] found that for air the diminution of the refraction was exactly proportional to the diminution of density when the temperature was raised, and Chappuis and Rivière[8] obtained the same result for cyanogen.

[1] Knops, Liebigs Ann., CCXLVIII., S. 175.

[2] Weegmann, Zeitschr. für physikal. Chemie, Bd. II., S. 218.

[3] Duet, Journal de Phys., 2e série, t. IV., pp. 389 et 477.

[4] Ketteler, Wied. Ann., Bd. XXX., S. 285; Bd. XXXIII., S. 353 u. 506; Bd. XXXV., S. 662.

[5] Mascart, Ann. Sc. de l'Ecole Norm. sup. (2), t. VI., p. 9, 1877; Comptes Rendus, t. LXXVIII., pp. 617, 679, 801, 1874; t. LXXXVI., pp. 321, 1182, 1878.

[6] v. Lang, Pogg. Ann., Bd. CLIII., S. 448, 1874.

[7] Benoit, Jour. de Phys. (2), t. VIII., p. 451, 1889.

[8] Chappuis et Rivière, Ann. de Chim. et de Phys. (6), t. XIV., p. 5, 1888.

Biot and Arago,[1] working between $-1.5°$ C. and $25°$ C., found that the "refractive power" $n^2 - 1$ was proportional to the density when d was computed by the formula, $d = \frac{d_0}{1 + \alpha t}$, α being the temperature coefficient of expansion. Their results, however, satisfy equally well the relation $\frac{n - 1}{d} = \text{const.}$

Lorenz[2] and Prytz[3] found that the relation $\frac{n^2 - 1}{n^2 + 2} \cdot \frac{1}{d}$ held fairly well for many substances though the liquid into the gaseous state —markedly better, in fact, than the relation $\frac{n - 1}{d}$ const. Some of their results are shown in the following tables:

	Liquid.		Gas.
	10°	20°	100°
Ether.	0.30264	0.30287	0.3068
Alcohol.	0.28042	0.28066	0.2825
Water.	0.20615	0.20608	0.2068
Chloroform.	0.17902	0.17909	0.1796

Mascart[4] conducted an extensive research on the effect of pressure on the index of refraction of air, using interference methods. Two tubes were filled with air under pressure, the one always a fixed amount higher than the other, and the relation between density and index of refraction was deduced from the number of fringes which passed as the air in the two tubes was brought to the same pressure. He found that the refraction $n - 1$ increased more rapidly than the pressure; the ratio increased about 1 per cent. when the pressure was increased to about 8 atmospheres. On taking into account variations of density in accordance with Regnault's formula, $d = AH(1 + BH)$, Mascart found that there were still discrepancies.

[1] Biot et Arago, Mem. de la prem. classe de l' Institut, t. VIII., p. 301, 1806; Mem. de l'Acad. des. Sciences, t. VII., p. 301, 1806; Gilberts Ann., Bd. XXV., S. 345, 1807; B. XXVI., S. 79, 1807.

[2] Lorenz, Wied. Ann., Bd. XI., S. 70, 1880.

[3] Prytz, Wied. Ann., Bd. XI., S. 104, 1880.

[4] Mascart, Ann. Sci. de l'Ecole Norm. sup. (2), t. VI., p. 9, 1877; Comptes Rendus, t. LXXVIII., pp. 617, 679, 801, 1874; t. LXXXVI., pp. 321, 1182, 1878.

which, however, he considered were within the limits of observational error.

Chappuis and Rivière[1] kept the pressure in one tube constan and measured the change dn in the index of refraction of the air in the other tube as its pressure was changed by an amount dH, Working with air and carbon dioxide up to twenty atmospheres they found that in every case the refractive power, $n - 1$, was proportional to the density.

Carnazzi,[2] however, working with a prism and a long beam of light, found that the ratio $\frac{n-1}{d}$ showed a decided increase in the case of air and hydrogen as the pressure was increased, but that the ratio decreased with increasing pressures in the case of carbon dioxide.

Since the work which had been done on the relation between density and index of refraction of air with a changing pressure did not appear to be entirely conclusive, an attempt has been made in the work described in this article to get some data on the subject. Interference methods naturally suggested themselves as the best for attacking the problem, but it was found advisable to modify the form of interferometer in common use. The pressure was measured in a simple way by applying the inverse principle of the McLeod gauge. The consideration of the investigation divides itself naturally into four parts: I. The pressure gauge, II. The optical arrangement, III. Observations, IV. Conclusions.

I. THE PRESSURE GAUGE.

The pressure gauge used is outlined in Fig. 1. AB is a steel rod about an inch in diameter, along the axis of which has been drilled a hole, C, about $\frac{1}{32}$ inch in diameter. D and E are steel stopcocks which were carefully ground with fine carborundum to fit as exactly as possible into tapering holes drilled through the rod AB. A piece of heavy copper pressure tubing of fine bore, screwed into one end of the rod and soldered in place, connected the pressure gauge to a large tank which contained air at the pressure to

[1] Chappuis et Rivière, Ann. de Chim. et de Phys. (6), t. XIV., p. 5, 1888.
[2] Carnazzi, Il Nuovo Cimento, 6, p. 385, 1897.

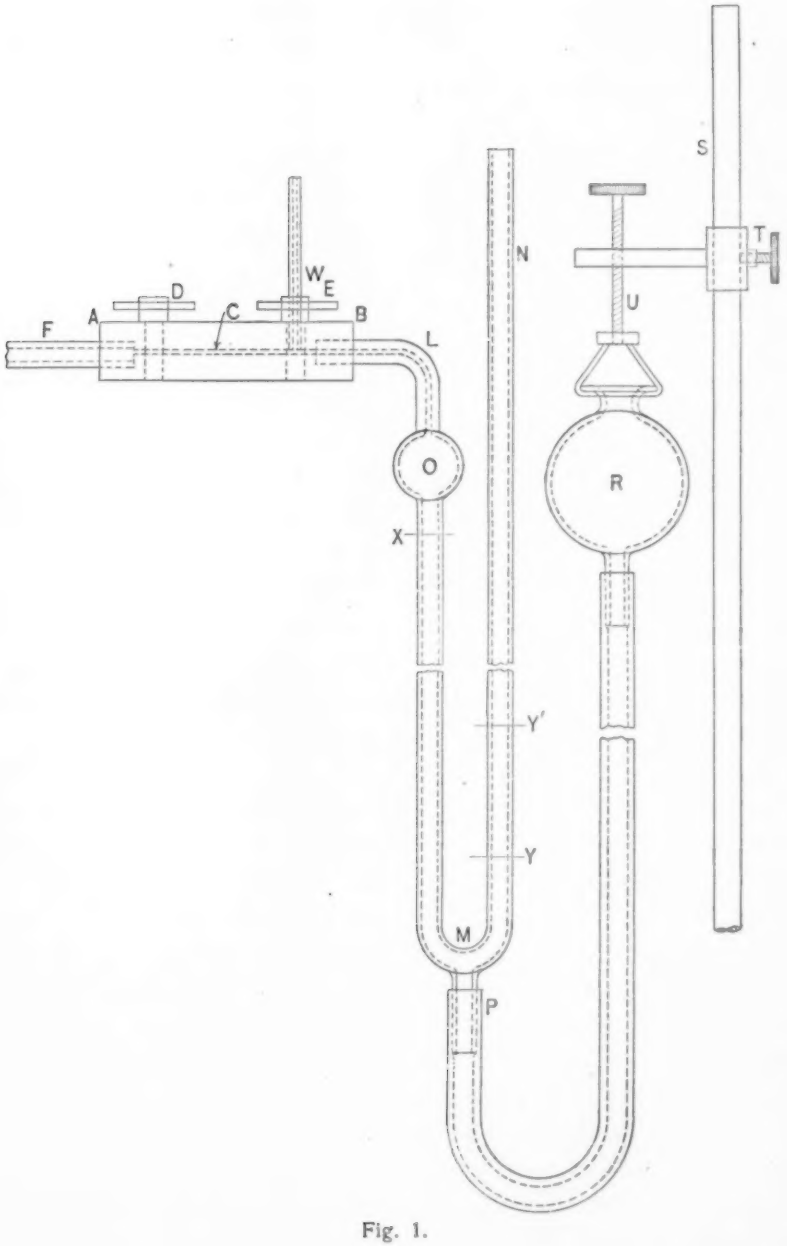


Fig. 1.

be measured. Into the other end of the rod *AB* was fastened one end of a glass tube *LMN* of the form shown in the figure. The bulb *O* had a capacity of about 10 c.c., and the tubes *OM* and *MN* had an internal diameter of about .5 cm. A bulb *R*, of about 500 c.c. capacity, for holding mercury, was connected with *P* by a piece of rubber tubing. The height of the mercury in the tubes *OM* and *MN* was regulated by sliding the clamp *T*, which held the bulb, up and down the rod *S*. A threaded rod *U*, which held the bulb, gave an opportunity for fine adjustment.

The stopcock *D* had a hole through it of small diameter in line with the hole *C*, through *AB*. The stopcock *E* was a three-way cock, having one hole through it in line with *C* and another half way through at right angles to the first. This stopcock was first set across so that the air could pass from *L* to the center of the stopcock and then out through the back of the rod *AB*, through a hole drilled to meet the hole *C*. A glass tube *W* was waxed into this hole to carry off the overflowing mercury.

As the bulb *R* was raised, mercury rose in the tube *NOL*, driving the air before it through the three-way cock *E*, and out into the tube at *W*. When the mercury began to rise in the tube at *W* the stopcock *E* was turned through 45°, thus closing all of its openings, and the bulb *R* was lowered until the mercury stood at some fixed point, as *X*, below the bulb *O*. The mercury then stood in the other arm of the tube at some point *Y*, almost a barometric column below *X*. A very high vacuum could not be expected in *O*, but by measuring with a cathetometer the difference of level between *X* and *Y* and subtracting this difference from the barometric height, the pressure of the gas left in *O* was found. The stopcock *D* was then opened and the small volume *C*, between the stopcocks, was allowed to fill with air at the pressure of the tank. Some little time was allowed for the pressure to adjust itself evenly, the stopcock *D* was then closed, and *E* was turned another 45°. This allowed the small volume of air in *C* at a high pressure to expand into the larger volume *O*, the mercury, of course, being forced down from *X*. The bulb *R* was then raised until the mercury came back exactly to its original position *X*, and a new reading was taken for the difference of level of the mercury in the two arms, the level in the second arm having been raised to some point *Y'*.

From these readings and the barometric height the pressure in the tank may be deduced in the following way: let V be the volume of the bulb and tube down to the point X, and call the pressure of the residual gas P. Let v' represent the volume of the space between the stopcocks, and p' the pressure of the gas in the tank. If the final pressure in the bulb, after the three-way cock has been opened and the mercury brought back to X, is represented by $\overline{P}$, the following equations will hold:

$$VP + v'p' = (V + v')\overline{P},$$

$$p' = \frac{V + v'}{v'}\overline{P} - \frac{V}{v'}P,$$

$$p' = k(\overline{P} - P) + P,$$

when

$$k = \frac{V + v'}{v'};$$

$$\therefore \frac{p'}{k} = \overline{P} - P + \frac{P}{k}.$$

Since $\overline{P} - P$ is equal to $Y' - Y$, this equation shows that the pressure p' in the tank is proportional to the change from Y to Y' if to this change be added the small quantity $\frac{P}{k}$ in which P stands for the residual pressure, which need never be more than a few mm., and k for the ratio of the total volume to the volume before expansion. This ratio need be found only roughly if it is desired to measure simply relative pressures. The value of the ratio in this experiment was about 30.

A further advantage was gained by the use of this form of pressure gauge. Since the density of a gas is not directly proportional to its pressure, if the pressure had been measured directly, the values of the density, computed from them by the simple law of Boyle, would have been too small. But if the gas is allowed to expand first by a definite amount and its pressure is measured at some smaller value, the departures from Boyle's law will be negligible, and the densities at the high pressures will be directly proportional to these lower pressures. This may be shown as follows:

$$M = V_0 D_0 = VD,$$

$$\frac{D}{D_0} = \frac{V_0}{V}.$$

But

$$V_0 P_0 (1 + aP_0 + bP_0^2) = VP(1 + aP + bP^2);$$

$$\therefore \frac{D}{D_0} = \frac{V_0}{V} = \frac{P(1 + aP + bP^2)}{P_0(1 + aP_0 + bP_0^2)};$$

where D_0, P_0, and V_0 represent the density, pressure, and volume under some standard condition, D, P, and V, the same quantities at some high pressure, and $(1 + aP + bP^2)$ is a factor which gives with a high degree of accuracy the departure from Boyle's law. But if the high pressure P is allowed to fall by expansion to some small pressure P', the following relations will hold:

$$VP(1 + aP + bP^2) = V'P'(1 + aP' + bP'^2),$$

and

$$P = \frac{V'P'}{V} \frac{(1 + aP' + bP'^2)}{(1 + aP + bP^2)};$$

$$\therefore D = \frac{D_0 V'P'(1 + aP' + bP'^2)}{VP_0(1 + aP_0 + bP_0^2)}.$$

Since the ratio $\frac{V'}{V}$ is constant, and since P_0, D_0, and the factor $(1 + aP_0 + bP_0^2)$ are constants, the result found above may be written

$$D = CP'(1 + aP' + bP'^2)$$

where C is a constant. If the pressures are less than one atmosphere, the factor $(1 + aP' + bP'^2)$ will not differ sensibly from unity, and the density D at the high pressure will be expressed as proportional to the small pressure P'. The readings on the pressure gauge will, then, be proportional to the densities at the high pressures and not to the high pressures themselves.

II. Optical Arrangement.

The air of which the index of refraction was to be found was confined in one of two similar stout tubes (Fig. 2), the ends of which were closed by pieces of plate glass. Two plates of steel, held to-

gether by bolts running from one end of the tubes to the other, pressed the pieces of plate glass tight against the ends of the tubes. Rubber washers were placed on each side of the glass plates. One of the tubes was connected by copper pressure tubing to the tank which contained the air under investigation, and to the pressure gauge.

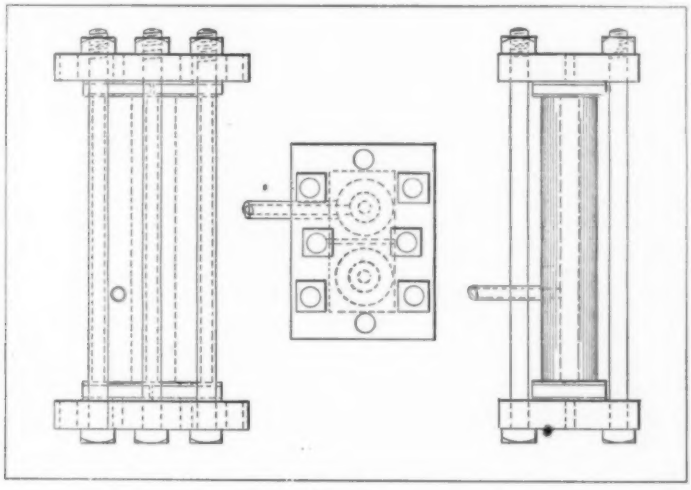


Fig. 2.

It was found convenient to arrange the interferometer mirrors in a way different from that in ordinary use, in a manner which is essentially a modification of Jamin's form. The objection to the latter is that the beams of light cannot be separated to any considerable extent except by the use of very thick plates. On the suggestion of Professor Michelson the mirrors were arranged as indicated in Fig. 3. *A* and *C* are "plane parallel" plates, each of which is coated with a thin film of silver, the plate *A* on the side nearer the source *S* and the plate *C* on the side nearer the telescope *T*. *B* and *D* are plane mirrors. Light coming from the source *S* strikes the thin film on *A*; half of the light is reflected to *B* and out through *C* to *T*. The rest of the light passes through *A* to *D* and is reflected to *C*. At *C* it is reflected, in part, by the thin film *C* to *T*. When the mirrors are properly adjusted, the conditions are right for interference, since the two parts of the original beam, when they arrive at *T*, have traveled nearly equal paths.

The tubes described above were placed between these mirrors, so that the light going from A to B passed down one tube, and that going from C to D passed down the other. When the apparatus was in adjustment and the fringes in view in the telescope, if the density of the air in one of the paths was changed, the fringes would shift.

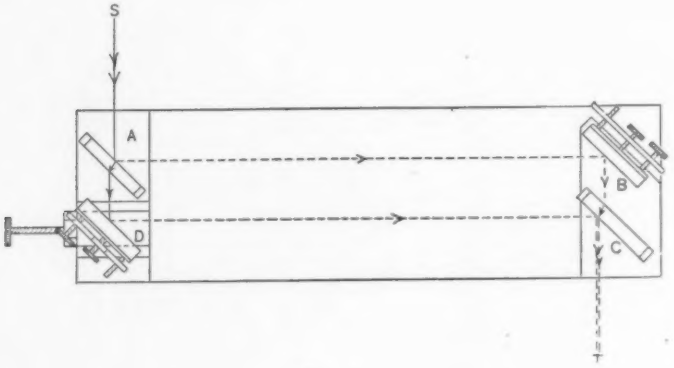


Fig. 3.

Let l equal the length of the tube in the path AB, λ the wave-length in a volume of the green light given out by incandescent mercury vapor in a vacuum tube, let n' and λ' be the index of refraction and wave-length, at a temperature t' and a pressure p'. Let N be the number of wave-lengths in the distance l in a vacuum, and N' the number at a temperature t' and a pressure p'. The following relations hold:

$$N = \frac{l}{\lambda},$$

$$N' = \frac{l}{\lambda'},$$

$$n' = \frac{\lambda}{\lambda'} = \frac{N'}{N},$$

$$n' - 1 = \frac{N'}{N} - 1 = \frac{N' - N}{N}.$$

Since N is constant this shows that the quantity $n' - 1$ under any conditions is proportional to $N' - N$, the number of fringes which

would pass on removing from the tube the air of index n', *i. e.*, the air at a temperature t' and pressure p'.

In practice it was, of course, impossible to remove all the air from the tube, but since the ratio $\frac{n-1}{d}$ was known to be very nearly constant, especially at low pressures, the pressure in the tube was simply allowed to fall to the pressure of the room, and the number of fringes was counted. The small additional number which would have passed if all of the air had been removed was computed in the following way, and added to the observed number. Let n'' and d'' be the index of refraction and density respectively of air at the temperature and pressure of the room, and let N'' be the number of wave-lengths in the tube under the same conditions. Let C represent the value of the constant ratio $\frac{n_0-1}{d_0}=\frac{n''-1}{d''}=C$. Then

$$n''=Cd''+1,$$

$$n''=\frac{N''}{N},$$

$$n''-1=\frac{N''-N}{N},$$

$$N=\frac{l}{\lambda_0 n_0};$$

$$\therefore N''-N=(n''-1)N,$$

$$=C\frac{d''l}{\lambda_0 n_0},$$

$$=C'd'',$$

where C' is a constant equal to $\left(\frac{n_0-1}{d_0}\right)\frac{l}{\lambda_0 n_0}$.

The quantity $N''-N$ is proportional to the density d'' and is equal to the number of fringes which would pass on removing the air at the pressure and temperature of the room, and is therefore the number of fringes to be added to the number counted while the

pressure was falling from p' to p'', in order to get the number which would have passed if the pressure had fallen from p' to 0.

III. OBSERVATIONS.

In the taking of observations the pressure was measured several times, and since only the small volume between the stopcocks was drawn off at each measurement, the pressure in the tank was not sensibly diminished. In the process of counting the number of fringes, the valve at the tank was opened and the tube between the interferometer mirrors and the connecting tubes were allowed to fill. The valve at the tank was then closed, and the union which joined the pressure tubing to the tank was gradually opened and the air in the apparatus allowed to escape. It was not at all difficult to regulate the escape so that the fringes would pass the cross hairs of the observing telescope at any desired rate. On filling the apparatus again and repeating the count, it was found that the number of fringes was slightly less than before, which meant that the pressure in the tank had been slightly reduced by drawing off enough air to fill the tubes. This amount of diminution was constant and it was not difficult to deduce the number of fringes which would have passed at the measured pressure, since this number corresponded to the number of the first count. The succeeding counts simply served as a check on the first one.

After a set of readings at one pressure had been taken, the air was allowed to escape from the tank into the room until the pressure had fallen to some desired point, when another set of readings was taken.

One set of readings, taken at the lowest pressure, is given below.

Temp. 23.8°. Bar. Ht. 74.785.

x	y	y'	$\overline{P}-P$	P	$\overline{P}-P+\frac{P}{K}$
103.5	29.250	40.345	11.095	.535	11.113
"	29.170	40.085	11.085	.455	11.100
"	29.100	40.205	11.105	.385	11.118
"	29.175	40.245	11.070	.460	11.085
"	29.160	40.240	11.080	.445	11.094
"	29.080	40.145	11.065	.365	11.077
"	29.085	40.155	11.070	.370	11.084

Mean—11.096.
Reduced to 0° C.—10.163.

$l = 15.024$ cm. $\lambda_0 = .00005461$. $n_0 = 1.0002928$. $N'' - N = 72.6$.

Fringes 249.0
248.8
248.6
248.4

$N' - N = 321.6$.

$$\frac{N' - N}{P - P + \frac{P}{K}} = 31.64.$$

A similar set of readings was taken for each of the other pressures, and the results are collected in the table below. In the first column is the approximate pressure in atmospheres; in the second column the pressure as read on the gauge, reduced to 0° C. The third gives the number of fringes, $N' - N$, and the fourth column gives the ratio

$$\frac{N' - N}{P - P + \frac{P}{K}}$$

which is proportional to $\frac{n-1}{d}$.

Atmos.	Press.	$N'-N$	$\frac{N-N'}{P-P+\frac{P}{K}}$	
4.	9.989	316.7	31.70	
	10.146	321.2	31.66	31.67
	10.163	321.6	31.64	
7.2	18.281	579.2	31.68	31.70
	18.365	582.7	31.72	
10.6	26.932	852.6	31.66	
14.2	35.990	1142.1	31.69	
19.2	48.780	1545.1	31.68	

IV. Conclusions.

There is probably not an error of more than .1 in any of the numbers of fringes. The errors in the pressures however may run as high as one or two tenths of 1 per cent. Since the variations in the values of the ratio at different pressures do not amount to more than .2 per cent., any apparent variations in the value of the ratio may be due to errors of observation. It seems, therefore, that if there is any departure from the law of Gladstone and Dale up

to twenty atmospheres in the case of air, this departure does not amount to more than about .1 per cent. The ratio between Lorentz's equation and that of Gladstone and Dale is so nearly equal to a constant, $\frac{2}{3}$ that an attempt to compare the two would be useless.

It is desired to express thanks to Professor Michelson, both for suggesting the experiment and for valuable advice and encouragement throughout the work. Thanks are due to Mr. F. B. Jewett for the drawings which accompany this article, and to Dr. Mann and Dr. Millikan for revision of the manuscript and checking of results.

MEASUREMENT OF THE INTERNAL RESISTANCE OF GALVANIC CELLS.

By C. H. Ayres, Jr.

IN the following paper it is proposed to set forth a new method of measuring electrolytic resistance, and to show that, in the case of certain galvanic cells, the *true* internal resistance is not a function of the current passing through the cell.

It is often questioned whether or not a galvanic cell has a fixed resistance, even at constant temperature; and many measurements have been made[1] which have been interpreted to show that the internal resistance decreases as the current passing through the cell increases. Where these measurements were made by steady-current methods, it has been impossibe to separate changes in electromotive force from changes in resistance, and in these cases the experimental results may be explained by assuming changes in electromotive force fully as well as by assuming changes in resistance. Where alternating currents have been used either the reristance has remained constant, or the variations have been much less pronounced than those shown by steady currents. Using an alternating current, F. Kohlrausch[2] found a value for the internal resistance which remained constant while the resistance in the bridge and the intensity of the current sent through the primary of the induction coil were varied between wide limits. Even with alternating currents Uppenborn[3] and Greef[4] found the resistance to depend on the current. Later, however, Haagn,[5] by a modification of Kohlrausch's method, found the internal resistance of galvanic cells to be independent of the current. The method here em-

[1] Streintz, Wied. Ann., 49, p. 571, 1893. Carhart, Physical Review, II., p. 392, 1895. Richarz, Wied. Ann., 47, p. 567, 1892.

[2] Pogg. Ann., Jubeland, p. 220, 1874. Pogg. Ann., 154, p. 1, 1875. Wied. Ann., 6, p. 1, 1878. Wied. Ann., 11, p. 653, 1880.

[3] Electrotech. Ztschr., 1891, p. 157.

[4] Greef Dissert. Marburg, 1895.

[5] Zeitschr. für Phys. Chem., 23, p. 97, 1897.

ployed, which is a bridge method, differs from the preceding in that the resistance and the capacity of the cell are separately, but in the final adjustment, simultaneously balanced, the self-induction being reduced to a negligible quantity. While attention is here directed to measurements of resistance, it may be noted that the method gives also the capacity of the cell.

THE METHOD.

The method here used is a modification of Kohlrausch's method, and is shown schematically in Fig. 1. In the arms AC and BC of a wheatstone bridge, the resistance r_1 and r_2 are in series with the condensers c_1 and c_2, respectively. AB is the bridge wire; I, an induction coil; and T the telephone used in obtaining a balance. For the sake of convenience, this arrangement shall be referred to as the "capacity bridge." a and b are the segments into which the slider, N, divides the bridge wire AB. When the induction coil is running and the system so adjusted that there is silence in the telephone, the double relation,

$$\frac{r_1}{r_2} = \frac{a}{b} = \frac{c_2}{c_1} \qquad \text{(A)}$$

is satisfied. If r_1 is the resistance to be measured, the capacities c_1 and c_2 may be fixed, and r_2, a, and b varied to satisfy the above relation. r_1 is then given by the equation

$$r_1 = \frac{a}{b} \cdot r_2. \qquad \text{(B)}$$

This method was chosen because by it the internal resistance of a galvanic cell may be measured when no steady current is flowing through the cell as well as when the cell is sending a current through any desired resistance.[1] Thus if a battery is inserted between A and c_1, no steady current can flow because of the condenser c_1, and hence, by means of an alternating current and telephone, the resistance may be obtained when the battery is delivering no current. The battery may then be shunted by a resistance S, the combined

[1] The method employed by Haagn, though different from the one here used, also afforded this advantage.

resistance of the battery and shunt measured, and, by the law of shunts, the internal resistance of the battery calculated. A second reason for the choice of this method will be shown farther on in this paper.

The induction coil I was a small one, such as in ordinarily used with a Kohlrausch bridge. The resistance of the primary coil was 2 ohms and that of the secondary about 30 ohms. The primary was interrupted by the device known as Neeft's hammer. The length of the spring carrying the hammer could be varied, its greatest length being twice its shortest. In order that the noise from the interrupter might not disturb the observer when using the telephone, the induction coil was placed at some distance from the remainder of the apparatus, in a room by itself.

The bridge wire AB (Fig. 1) was a meter long, and, on different occasions, had a resistance of from 3 to 5 ohms.

In the first series of experiments, the condensers c_1 and c_2 were made of paper and their capacities were varied from 10 to 40 microfarads. In a later series of experiments, mica condensers of 30 microfarads each were used.

Several telephones were always at hand, and that one was chosen which seemed best suited to the conditions of the experiment under consideration. The one most used was a Swiss telephone of about 80 ohms resistance. Sometimes, however, an American telephone, having a resistance of about 1 ohm, was used.

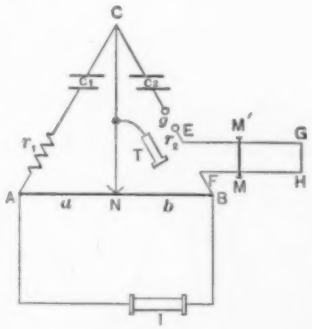


Fig. 1.

The method of constructing the variable resistance, r_2, may be readily explained by reference to Fig. 1. Between C and E is a gap, g, into which a resistance coil of any desired size may be inserted. $EGHF$ is a manganine wire, about two meters in length and of about 1.25 ohms resistance, fastened at E and F, and stretched over the two posts G and H. MM' is a trough carrying a drop of mercury connecting the two parts, EG and HF, of the wire. The resistance in g, together with the resistance of EM' and MF, makes

the resistance r_2. r_2 could be varied, therefore, by changing the resistance in g and by sliding MM' along the wire. By amalgamating the wire before each experiment, an excellent contact between the mercury and the wire was secured.

The resistance to be measured was inserted in the arm AC between A and c_1, and its value calculated equation from (B). In order to determine the resistance r_2, it was thrown from the arm of the capacity bridge into an arm of a wheatstone bridge, where it was compared with a standard resistance by means of a steady current and galvanometer. Fig. 2 shows the connections of the two bridges. G represents the galvanometer and E the cell used with the second bridge. The transfer of r_2, from one bridge to the other was effected by means of a two-way

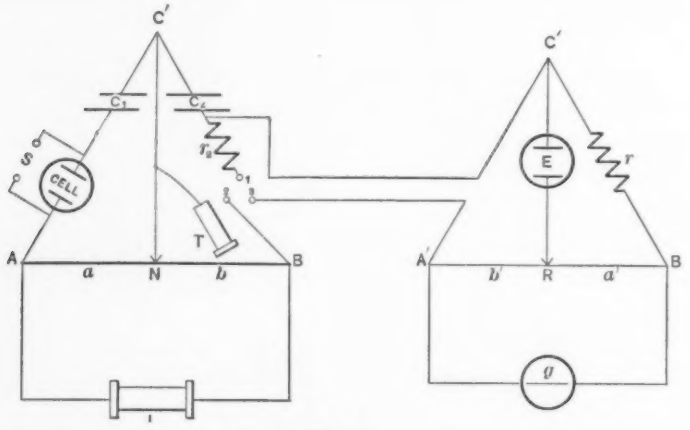


Fig. 2.

switch, which consists of three mercury wells and a heavy copper strap. When wells 1 and 2 were connected by the strap, r_2 was in the capacity bridge; and when the strap connected wells 1 and 3, r_2 was in the second or steady-current bridge. This second bridge was made entirely of copper, in order to reduce the thermo-electric effects to a minimum; but as r_2 was manganine, not copper, and thermo-electric currents might therefore be expected, the galvanometer was connected to the two ends of the bridge wire, its circuit being always closed, while the battery was placed across the bridge, its circuit being open, except for short intervals of time.

In this way, all trouble from thermoelectric effects was avoided.

It is not a difficult operation to satisfy the double relation

$$\frac{r_1}{r_2} = \frac{a}{b} = \frac{c_2}{c_1}. \qquad (A)$$

One has merely to move the slider, N, along the bridge wire until a minimum is found, then vary r_2 so as to improve the minimum; then again move the slider, and so on until silence is obtained. Silence having been obtained, r_2 is put into the copper bridge and compared with a standard resistance. If r is the resistance against which r_2 is balanced in the copper bridge, a' and b' the segments of the bridge wire, then

$$r_2 = \frac{b'}{a'} \cdot r$$

and, by equation (B),

$$r_1 = \frac{b'}{a'} \cdot r \cdot \frac{a}{b} \qquad (C)$$

a and b being the segments of the wire on the capacity bridge.

When metallic resistances, free from self-induction, are used in the bridge, perfect silence is obtainable in the telephone. This is not always the case when a liquid resistance is inserted in one arm of the bridge. When polarization takes place in the liquid cell, a disturbing effect is produced. This effect we shall refer to as the capacity effect of the cell, for it has been shown [1] that a polarized cell may be regarded as a resistance in series with a capacity. When such an effect exists, in the arrangement of the bridge here used, it is combined with the capacity of the condenser with which it is in series, and causes no error in the determination of resistance. If a current of a single frequency were used in working the bridge, it would always be possible to obtain silence in the telephone, but when the current is made up of vibrations of different frequencies, only a minimum sound is obtainable, because the capacity effect of a cell is different for different frequencies and it is impossible to balance for all frequencies at once.[2] Suppose the relations (A) are satisfied for metallic resistance r_1 and r_2. If any other metallic re-

[1] Varley, Phil. Mag., 4, 41, p. 310, 1871. Kohlrausch and Holborn, "Leitvermögen der Elektrolyte," p. 67.

[2] M. Wien, Wied. Ann., 42, 593, 1891; 47, 626, 1892; 58, 37, 1896; 59, 267, 1896.

ristance r_1' is substituted for r_1, silence may again be obtained by varying the resistance r_2, leaving the slider, N, unmoved, for c_1 and c_2 remain unchanged and

$$\frac{a}{b} = \frac{c_2}{c_1}.$$

Now suppose the resistance r_1' to be a liquid resistance in which polarization takes place. The capacity of the arm AC will no longer be c_1, but c_1', a combination of c_1 and the capacity effect of the cell. It will not now be possible to obtain a minimum by varying r_2 only, but the slider N must also be moved, for the segments a and b of the bridge wire must satisfy the relation

$$\frac{a}{b} = \frac{c_2}{c_1'}.$$

Even in case the capacity effect of the cell causes a considerable displacement of the position of the slider corresponding to a minimum, the resistance of the cell may be accurately measured, for a minimum is determined by the double relation

$$\frac{a}{b} = \frac{c_2}{c_1'} = \frac{r_1'}{r_2},$$

and r'_1 is given by the equation

$$r_1' = \frac{a}{b} \cdot r_2.$$

In this respect, the form of the bridge under consideration affords an advantage over the Kohlrausch bridge in which a displacement of the minimum causes an error in the determination of resistance. This advantage formed the second reason for the development of the method.

RESULTS AND DISCUSSIONS.

Experiments with Paper Condensers.—The cell to be measured was always inserted in the arm AC so that its resistance corresponded to that which has been called r_1. Care was taken to make the resistance of the connecting wires in the arms AC and BC as small as possible. In order to determine whether or not these wires caused serious error in the measurement of resistance, standard coils were measured. The results showed that resistance varying from

one to three ohms could be measured with an accuracy of a few tenths of one per cent. The results given here are all above one ohm, and are probably accurate to within .5 per cent. Allowance was made for the resistances of the end connections of the bridge wires. In the copper bridge, in which the wire was of very low resistance, the end connection at A' (Fig. 2) was equivalent to the resistance of .7 cm. of the bridge wire, while that at B' was equivalent to .8 cm. On the capacity bridge, the connection resistance at A was equivalent to .19 cm. of the bridge wire, and that at B to .33 cm. In the experiments with mica condensers, the connection resistances were so reduced as to be negligible.

The first cell measured was a modified Daniel cell, composed of copper in sulphate of copper and zinc in sulphate of zinc. The copper electrode and the sulphate of copper were placed in a large glass jar, and into the solution of copper sulphate was sunk a porous cup containing the zinc and the zinc sulphate. The cell is shown in the adjoining figure, and will be referred to by its commercial name, "Excello Cell." In the tables of observations, given below, T represents the temperature of the cell, as given by a thermometer, the bulb of which was placed in the porous cup; N and R denote the positions of the sliders on the capacity and copper bridges, respectively; S denotes the resistance by which the cell was shunted, and b is the value found for the resistance of the cell. When no shunt is used, the symbol ∞ is placed in column S.

Fig. 3.

It is noticeable that the setting of the slider was practically the same for all these measurements. This shows that if the capacities c_1 and c_2 of the paper condensers changed at all during the experiment, the changes were such that the ratio c_2/c_1 remained constant.

Next the cell was shunted by resistances varying from 100 to 3 ohms, and its resistance measured.

In these experiments, the alternating current in the wires leading from the induction coil was less than .005 ampère, of which only a small part went through the cell. Thus the current used in

$C_1 = C_2 = 20$ *microfarads.* *August* 2, 1900.

T	N	R	S	b
25.6	51.20	38.40	100	1.700
25.6	51.20	37.98	∞	1.704
25.6	51.20	38.27	100	1.708
25.6	51.20	38.10	∞	1.693
25.6	51.20	38.47	100	1.698
25.6	51.20	38.03	∞	1.698
25.6	51.20	38.40	100	1.700
25.6	51.20	38.00	∞	1.700
25.5	51.20	38.41	100	1.700
25.5	51.20	38.02	∞	1.698
25.5	51.20	38.41	100	1.700
25.5	51.20	37.94	∞	1.704
25.5	51.20	38.40	100	1.700
25.4	51.20	37.98	∞	1.704
25.4	51.20	38.44	100	1.698
25.4	51.20	37.97	∞	1.704
25.4	51.20	38.44	100	1.698
25.4	51.21	37.88	∞	1.709
25.4	51.21	38.42	100	1.700
25.4	51.22	38.00	∞	1.700

Mean of open circuit resistance = 1.701 ohms. Mean of closed circuit resistance = 1.700 ohms.

$C_1 = C_2 = 20$ *microfarads.* *August* 3, 1900.

T	N	R	S	b
23.7	51.20	39.57	100	1.613
23.7	51.20	39.16	∞	1.611
23.7	51.20	39.85	50	1.616
23.7	51.20	39.15	∞	1.611
23.7	51.20	40.10	40	1.616
23.7	51.20	39.20	∞	1.608
23.7	51.20	40.52	30	1.604
23.7	51.20	39.18	∞	1.610
23.7	51.20	41.16	20	1.603
23.7	51.20	39.15	∞	1.612
23.7	51.20	42.81	10	1.611
23.7	51.20	39.21	∞	1.607
23.7	51.10	49.52	3	1.604
23.7	51.20	39.25	∞	1.605

measuring the resistance was not merely alternating, but also extremely small in comparison with the steady current.

The following table contains the results obtained on LeClanche, Daniell, and Excello cells:

	$c_1 = c_2 = 20$ m.f.			$c_1' = c_2 = 40$ m.f.		
	Excello Cell No. 1.	Excello Cell No. 2.	Excello Cell No. 3.	LeClanche Cell.	Daniell Cell No. 1.	Daniell Cell No. 2.
∞	1.603			1.584	1.613	1.184
100	1.602	1.325	1.092	1.540		
50	1.603					
40	1.603		1.072	1.575		
30	1.603		1.095	1.561	1.611	
20	1.604		1.095	1.551	1.620	
10	1.600	1.321	1.096			1.181
3	1.603		1.110			

Figures for Excello and LeClanche cells represent single observations, while those for the Daniell cells are means of a number of observations taken in succession. It was impossible to obtain a reliable minimum when low shunts were used on the LeClanche cells. In all these experiments, the minimum was sharp and its position was readily found. In the case of the Excello cells, it was found that if the zinc electrode was kept clean and well amalgamated the minimum was not only sharp, but symmetrical, as is the case when metallic resistances are used. Moreover, when the zinc electrode of these cells was clean, the position of the minimum was practically the same as for metallic resistances. This shows that there was no disturbing effect due to polarization. In the case of the LeClanche and Daniell cells, the minimum was displaced from 1 to 2 mm. from that given by metallic resistances. Under these circumstances, of course, the effect being different for different frequencies, the minimum was not symmetrical.

Experiments with Mica Condensers.

The mica condensers used in this series of experiments had a capacity of 30 microfarads each. The resistance of the wires connecting the various sections was very small and as nearly as possible the same in both condensers. Before beginning these experiments, the two bridges were thoroughly overhauled and the resistances of the end connections of the bridge wires were made

negligible. Measurements on known metallic resistance, made to test the accuracy of the bridge, gave results correct to one-twenty-fifth per cent. In the experiments with this apparatus, the cells were measured first with no shunt and then with a shunt of 1 ohm. In order to keep the condition of the cells as steady as possible, they were shunted by a resistance of 10 ohms during the time between the different observations.

The table below contains the results obtained from five different cells. The zinc electrode of Daniell cell No. 1 consisted of a rod about three inches long and three-sixteenths of an inch in diameter. The zinc electrode of Daniell cell No. 2 was in the shape of a rectangle about 10 centimeters long and 2 centimeters wide. The electrode in the Excello cells, as well as those of the Daniell cells given in the first table, were of the size ordinarily used in these cells.

S.	Excello Cell No. 1.	Excello Cell No. 2.	Daniell Cell No. 1.	Daniell Cell No. 2.	Columbia Dry Cell.
∞	.6133	.5838	2.650	2.044	.1151
1	.6123	.5845	2.651	2.027	.1151
∞	.6138	.5843	2.656	2.036	.1157
1	.6117	.5855	2.652	2.052	.1151
∞	.6112	.5863	2.659	2.036	.1147
1	.6117	.5863	2.651	2.027	.1158

The results given above show that the resistance was the same whether the cell was in open or closed circuit, within high limits of accuracy. However, it is to be noted that the condition of a cell, of course, depends upon its previous history, among other things upon any prolonged current that may have been flowing, insofar as this actually alters the cell. What is here shown is that the resistance of a cell is not a function of the current strength at the time, a result contrary to the conclusions of most other observers.

ON THE THEORY OF THE BRIDGE.

Thus far we have considered only the apparatus used and the results obtained in actual experiment. It will now be of interest to consider the relations between the current in the telephone and the magnitude of the various quantities used in the bridge, for from these relations we may see what conditions are necessary for accurate

work. Our arrangement of the bridge is represented in Fig. 1. In the arms AC and BC the resistance r_1 and r_2 are in series with the capacities c_1 and c_2, respectively. T denotes the telephone, and I the induction coil used with the bridge. Let r_3 and r_4 be the resistances of the segments AN and NB of the bridge wire AB. For the sake of clearness, the various quantities will be considered under separate heads.

Capacity.—In order to form an idea of how much capacity must be used to render accurate measurements of resistance possible, it is necessary to write down the expression for the current in the telephone. For the sake of simplicity, assume the resistance and self-induction of the main branch, AIB, to be so large that changes in the capacities c_1 and c_2 do not materially alter the current supplied to the bridge. If the current in the main branch is assumed to be

$$\cos(nt) = \tfrac{1}{2}(e^{int} + e^{-int}),$$

the current in the telephone at any time may be found by taking one-half the sum of the currents in the telephone when currents e^{int} and e^{-int} are separately taken as the currents in the main branch.

If there is a current e^{int} in the main branch, the current in the telephone is given by

$$Z' = e^{int}\frac{a_2a_3 - a_1a_4}{(a_2 + a_4)(a_1 + a_3) + a(a_1 + a_2 + a_3 + a_4)} \qquad (1)$$

where

$$a_1 = r_1 + \frac{1}{inc_1},$$

$$a_2 = r_2 + \frac{1}{inc_2},$$

$$a_3 = r_3,$$

$$a_4 = r_4,$$

and

$$a = r + inL,$$

r being the resistance, and L the inductance of the telephone. Putting these values for the a's in equation (1), we obtain,

$$Z' = \frac{e^{int}(A + iB)}{1 - C + iD} \qquad (2)$$

where

$$A = n^2c_1c_2(r_1r_4 - r_2r_3),$$

$$B = n(r_3c_1 - r_4c_2),$$

$$C = n^2k_1k_2c_1c_2 + n^2rc_1c_2(k_1 + k_2) + n^2L(c_1 + c_2),$$

$$D = n[K_2c_2 + k_1c_1 + r(c_1 + c_2) - n^2L(k_1 + k_2)c_1c_2],$$

$$k_1 = r_1 + r_3$$

and

$$k_2 = r_2 + r_4.$$

Multiplying the numerator and denominator of (2) by $(1 - C) - iD$, we obtain,

$$Z = e^{int}\frac{(A + Bi)[(1 - C) - iD]}{(1 - C)^2 + D^2}$$

or

$$Z' = e^{int}(A'' + iB'') \tag{3}$$

where

$$A'' = \frac{A(1 - C) + BD}{(1 - C)^2 + D^2} \tag{4}$$

and

$$B'' = \frac{B(1 - C) - AD}{(1 - C)^2 + D^2} \tag{5}$$

If e^{-int} is taken as the current in the main branch, the current in the telephone is given by

$$Z'' = e^{-int}(A'' - iB''),$$

and the current in the telephone corresponding to a current cos (nt), in the main branch, is

$$Z = \tfrac{1}{2}(Z' + Z'')$$

$$= A''\frac{e^{int} + e^{-int}}{2} + B''\frac{e^{int} - e^{-int}}{2}i$$

$$= A'' \cos(nt) - B'' \sin(nt)$$

whence

$$Z = \sqrt{A''^2 + B''^2}\cos(nt - \varphi) \tag{6}$$

where

$$\tan\varphi = -\frac{B''}{A''}.$$

When $r_1/r_2 = r_3/r_4$, we have $A = 0$, and when $c_1/c_2 = r_4/r_3$, $B = 0$. When both A and B are zero, A'' and B'' are zero; and, therefore, by (6), $Z = 0$. Now if it is desired to measure resistances there must be a disturbance in the telephone whenever $r_1/r_2 \neq r_3/r_4$, even if $c_1/c_2 = r_4/r_3$; that is, even if $B = 0$. It is, therefore, desirable to study the relations that must exist between the resistances, capacity, and inductance in order that the current in the telephone may be detected as long as the resistances do not satisfy the condition $r_1/r_2 = r_3/r_4$. The work is much simplified, while its usefulness is not at all impaired, by the assumption $B = 0$. If $B = 0$,

$$A'' = \frac{A(1 - C)}{(1 - C)^2 + D^2},$$

and

$$B'' = \frac{-AD}{(1 - C)^2 + D^2},$$

hence, if the maximum value of Z is called $\bar{Z}$, we may write

$$\bar{Z} = \sqrt{\left[\frac{A(1 - C)}{(1 - C)^2 + D^2}\right]^2 + \left[\frac{-AD}{(1 - C)^2 + D^2}\right]^2},$$

or

$$\bar{Z} = A[(1 - C)^2 + D^2]^{-\frac{1}{2}}. \qquad (7)$$

Now

$$A = n^2 c_1 c_2 (r_1 r_4 - r_2 r_3);$$

therefore, A increases as the product $c_1 c_2$, and, if C and D are small in comparison with unity as is the case when $c_1 c_2$ are small, $\bar{Z}$ will increase as the product $c_1 c_2$. In this case evidently $\bar{Z}$ may be increased by increasing r_3 and r_4, *i. e.*, by increasing the resistance of the bridge wire. Let us now assume that $c_1 = c_2$, as was the case in all the experiments made with this arrangement of the bridge, and write

$$P = \frac{1 - C}{c_1^2},$$

and

$$Q = \frac{D}{c_1^2},$$

or

$$P = \frac{1}{c_1^2} - \left[n_2 k_1 k_2 + n^2 r(k_1 + k_2) + \frac{2n^2 L}{c_1}\right] \qquad (8)$$

and

$$Q = n\left[\frac{k_1 + k_2 + 2r}{c_1} - n^2L(k_1 + k_2)\right]. \tag{9}$$

Then equation (7) becomes

$$\bar{Z} = \frac{n^2c_1^2}{c_1^2}(r_1r_4 - r_2r_3)\left[P^2 + Q^2\right]^{-\frac{1}{2}},$$

or

$$\bar{Z} = n^2(r_1r_4 - r_2r_3)\left[P^2 + Q^2\right]^{-\frac{1}{2}}. \tag{10}$$

Consider c_1 the only variable in the right hand side of equation (10). Z is largest for that value of c_1 which makes $[P^2 + Q^2]^{\frac{1}{2}}$ smallest, or for that value which makes

$$D_{c_1}P \cdot P + D_{c_1}Q \cdot Q = 0 \tag{11}$$

Putting in (11) the values of P and Q given by (8) and (9), we obtain,

$$ac_1^3 - a_1c_1^2 + a_2c_1 - a_3 = 0 \tag{12}$$

where

$$a = n^4L(k_1^2 + k_2^2)$$
$$a_1 = n^2[k_1^2 + k_2^2 + 2r(k_1 + k_2) + 4r^2] + 4n^4L_2$$
$$a_2 = 6n^2L$$

and

$$a_3 = 2.$$

It is evident that equation (12) always has at least one real, positive root. Hence for any given resistances and inductance there is always at least one value for c_1 that makes $\bar{Z}$ a maximum.

Let us now study the variations of $\bar{Z}$ with c_1 for the special case in which $c_1 = c_2$, $r_3 = r_4 = 1$, $r_1 = 1 + x$, $r_2 = 1$, $r = 1$, $L = \frac{1}{2}\,10^{-4}$, and $n = 10^3$, these values being chosen because they are of the same order of magnitude as those that were used occasionally in actual experiment. x is here the amount by which r_1 deviates from satisfying the relation $r_1/r_2 = r_3/r$. For this case,

$$k_1 = 2 + x;$$
$$k_2 = 2;$$
$$A = 10^6c_1^2x;$$
$$C = 10^6(8 + 3x)c_1^2 + c_1 10^2$$

and

$$D = 10^3\left[6c_1 + xc_1 - 50(4 + x)c_1^2\right].$$

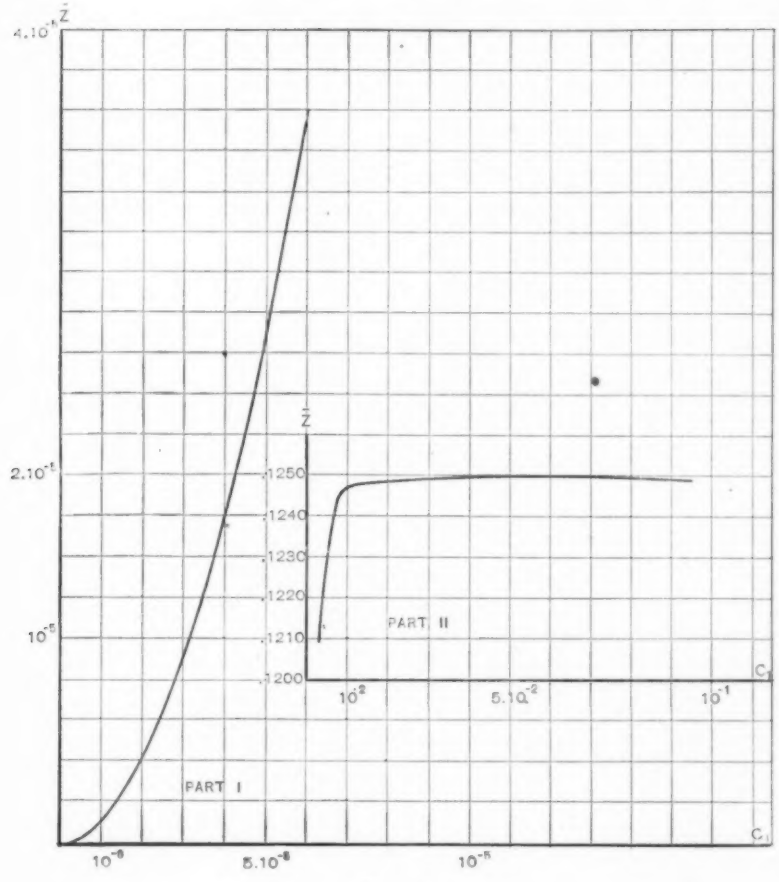


Fig. 4.

If x is small in comparison with unity, we may write,

$$C = 8 \cdot 10^6 \cdot c_1^2 + c_1 \cdot 10^2,$$

and

$$D = 10^3 \left[6c_1 - 2 \cdot 10^2 \cdot c_1^2\right].$$

This makes, by (8) and (9),

$$P = \frac{1}{c_1^2} - 10^2 \left[10^4 \cdot 8 + \frac{1}{c_1}\right]$$

and

$$Q = 10^8 \left[\frac{6}{c_1} - 2 \cdot 10^2\right].$$

By equation (10),

$$Z = 10^6 x[P^2 + Q^2]^{-\frac{1}{2}}. \qquad (13)$$

To find the value of c_1 for which $\bar{Z}$ is a maximum, we substitute for P and Q in the equation,

$$D_{c_1}P \cdot P + D_{c_1}Q \; Q = 0.$$

This gives,

$$4 \cdot 10^8 c_1 - 2001 \cdot 10^4 c_1^2 + 3 \cdot 10^2 c_1 - 2 = 0. \qquad (14)$$

By Sturm's theorem, equation (14) is seen to have but one real root; this root lies between $c_1 = 10^{-1}$ and $c_1 = 10^{-2}$. Hence there is but one value of c_1 which renders $\bar{Z}$ a maximum. The relation between c_1 and $\bar{Z}$ is shown by Fig. 4. This curve is given in two parts, each part having a scale of its own. Part I. shows the rapid increase of $\bar{Z}$ with c_1, when c_1 is small. Part II. shows the maximum value of $\bar{Z}$, which corresponds very nearly to $5 \cdot 10^{-2}$ farads. Between Parts I. and II. a portion of the curve, having no peculiar points, is omitted. The table below gives a few values of c_1, together with the corresponding values of $\bar{Z}$.

c_1	$\bar{Z}$	c_1	$\bar{Z}$
0	0	10^{-2}	$.1248x$
10^{-6}	$10^{-6}x$	2.10^{-2}	$.1249x$
2.10^{-6}	$4.10^{-6}x$	5.10^{-2}	$.1250x$
10^{-5}	$10^{-4}x$	10^{-1}	$.1249x$
10^{-4}	$9.2x10^{-3}$	1	$.1249x$
10^{-3}	$.109x$	∞	$.1249x$

The table shows that $\bar{Z}$ at the maximum is only slightly greater than the limit approached as c_1 increases without limit; that is, $\bar{Z}$ is never much larger than it would be if resistances only were used in the bridge. The value of c_1 corresponding to a maximum of $\bar{Z}$ is, in the case under consideration, exceedingly large. It is, therefore, fortunate that accurate work may be done long before the best condition is attained. With a bridge wire of two or three ohms resistance, and with capacities of from twenty to forty microfarads, a resistance as low as half an ohm may be measured with an accuracy of .5 per cent.

If in place of the telephone, we have a non-inductive conductor of resistance, r, across the bridge,

$$P = \frac{1}{c_1^2} - n^2 k_1 k_2 - n^2 r(k_1 + k_2),$$

and

$$Q = n[k_1 + k_2 + 2r]c_1^{-1}.$$

In this case,

$$Dc_1 P \cdot P + Dc_1 Q \cdot Q = 0,$$

becomes

$$c_1^2[4r^2n^2 + 2rn^2(k_1 + k_2) + n^2(k_1^2 + k_2^2)] + 2 = 0.$$

Since the roots of this equation are always imaginary, there is no value of c_1 for which $\bar{Z}$ is a maximum. In fact,

$$\bar{Z} = n^2(r_1 r_4 - r_3 r_2)\left[\frac{1}{c_1^4} + \frac{G}{c_1^2} + H\right]^{-\frac{1}{2}}$$

where

$$G = n^2(k_1^2 + k_2^2) + 4n^2r^2$$

and

$$H = n^4 k_1 k_2 + n^4 r^2 (k_1 + k_2)^2 + 2n^4 r k_1 k_2 (k_1 + k_2).$$

And it is evident that $\bar{Z}$ increases when c increases, as would naturally be expected.

Inductance.—That there is a value of the inductance, L, of the telephone which renders $\bar{Z}$ a maximum when the resistances and capacity in the bridge are fixed, may be seen by placing the derivative, with respect to L, of the right-hand side of the equation (10) equal to zero. This gives the equation,

$$D_L P \cdot P + D_L Q \cdot Q = 0,$$

an equation which is linear as regards L. Putting in the values of P and Q, we obtain,

$$L = \frac{c_1(k_1^2 + k_2^2) + \frac{2}{n^2 c_1^2}}{4 + n^2(k_1 + k_2)^2 c_1^2}. \qquad (14)$$

From this equation, it is evident that the smaller c_1, the larger the inductance required to make $\bar{Z}$ a maximum. That the resistance of the telephone does not enter into this expression for the inductance is not surprising, for we have assumed that we could vary the inductance of the telephone without varying its resistance.

$$k_1 = k_2 = 2,$$

$$c_1 = 4 \cdot 10^{-5},$$

and

$$n = 10^3,$$

we find

$$L = .0124 \text{ henry.}$$

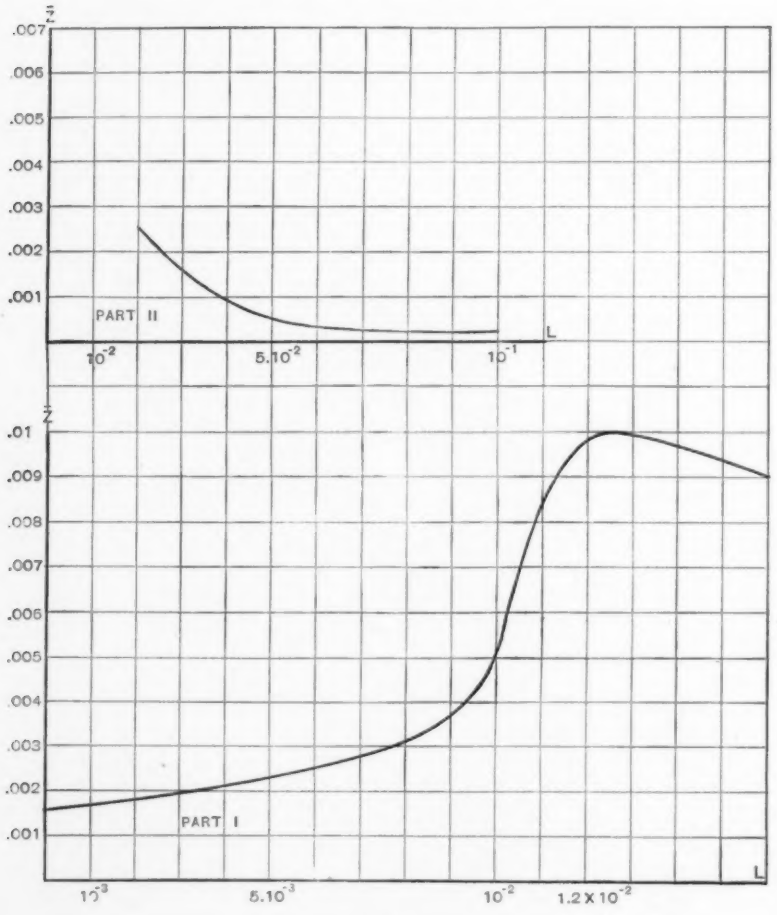


Fig. 5.

Fig. 5 shows the relation between L and $\bar{Z}$ as calculated from equation (13) on the assumption that

$$r_2 = r_3 = r_4 = r = 1 \text{ ohm.}$$

$$r_1 = 1 + x,$$

$$n = 10^3,$$

and

$$c_1 = 40 \text{ microfarads.}$$

This curve is given in two parts, having different scales. Part I. contains the maximum and is of chief interest. There is a small and unimportant part of the curve omitted between Parts I. and II. The following are a few values of L, with the corresponding values of $\bar{Z}$.

L	$\bar{Z}$	L	$\bar{Z}$
0	.00157x	$1.24.10^{-2}$	.0100x
10^{-5}	.00158x	2.10^{-2}	.00257x
10^{-4}	.00158x	5.10^{-2}	.000506x
10^{-3}	.00170x	10^{-1}	.000230x
10^{-2}	.00525x	∞	.0000

Frequency.—In order to see if, by varying n, keeping the resistances and inductance constant, a value of n can be found for which $\bar{Z}$ is a maximum, it will be well to write equation (7) in the form,

$$\bar{Z} = c_1^2(r_1r_4 - r_2r_3)[P_1^2 + Q_1^2]^{-\frac{1}{2}} \qquad (15)$$

where

$$P_1 = \frac{1}{n^2} - [k_1k_2c_1 + rc_1^2(k_1 + k_2) + 2Lc_1],$$

and

$$Q = \frac{k_1c_1 + k_2c_1 + 2rc_1}{n} - nL(k_1 + k_2)c_1^2,$$

c_1 being taken equal to c_2, as before.

To make $\bar{Z}$ a maximum, we make

$$D_nP_1 \cdot P_1 + D_nQ_1 \cdot Q_1 = 0. \qquad (16)$$

Substituting the values of P_1 and Q_1 in this equation, we obtain, after reduction,

$$n^6L^2(k_1 + k_2)^2c_1^4 + n^2\,[4Lc_1 - 4r^2c_1^2 - 2rc_1^2(k_1+k_2) - k_1^2c_1^2 - k_2^2c_1^2] - 2 = 0.$$

Or, if we let $n_1 = n^2$

$$n_1^3L^2(k_2 + k_1)^2c_1^4 + n[4Lc_1 - 4r^2c_1^2 - 2rc_1^2(k_1 + k_2) - k_1^2c_1^2 - k_2^2c_1^2] - 2 = 0. \qquad (17)$$

If, for example, we take

$$c_1 = 3 \cdot 10^{-5},$$

$$L = \tfrac{3}{2}\ 10^{-4},$$

and

$$k_1 = k_2 = 2,$$

equation (17) reduces to

$$n_1^3 = \frac{10^{28}}{145\ 8},$$

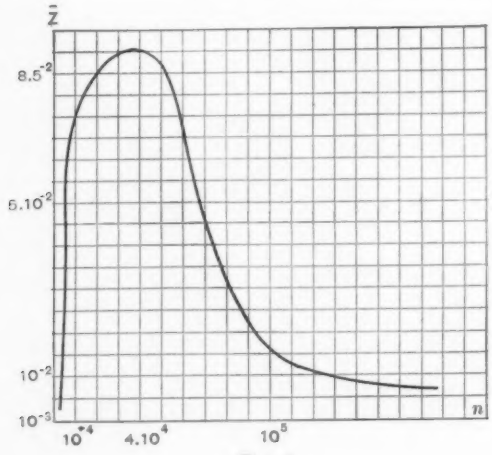


Fig. 6.

or

$$n_1^3 = \frac{10^{27}}{146};$$

hence

$$n_1 = \frac{10^9}{(146)^{\frac{1}{3}}}$$

and

$$n = \frac{10^{\frac{2}{3}}}{(146)^{\frac{1}{3}}},$$

$$n = 4 \cdot 10^4.$$

And the number of complete vibrations per second is given by $\frac{n}{2\pi}$, or $\frac{2.10^4}{\pi} = 6366.$

Fig. 6 shows the relation between $\bar{Z}$ and n as calculated from equation (13); it being assumed that $k_1 = k_2 = 2$,

$$c_1 = 3 \cdot 10^{-5},$$

and

$$L = \tfrac{3}{2}\, 10^{-4}.$$

The following table shows a few of the values of n, together with the corresponding values of $\bar{Z}$.

n	$\bar{Z}$	n	$\bar{Z}$
0	0	10^5	$1.6.10^{-2}x$
10^2	$9.10^{-6}x$	10^6	$1.7.10^{-3}x$
10^3	$9.10^{-4}x$	10^7	$1.7.10^{-4}x$
10^4	$7.10^{-2}x$	10^8	$1.7.10^{-5}x$
4.10^4	$8.6.10^{-2}x$	∞	0

It is to be noticed that $\bar{Z}$ increases rapidly at first as n increases, reaching a maximum when n is $4 \cdot 10^4$, and then decreases as n increases, approaching the value of zero as n increases without limit.

The above treatment shows how the accuracy of measurement depends upon the relations between inductance, capacity, resistance, and frequency of current used in the bridge.

ON THE DIELECTRIC CONSTANT OF DILUTE ELECTROLYTIC SOLUTIONS.

BY A. DE FOREST PALMER, JR.

IN 1892 E. Cohn[1] published the results of a series of determinations of the refractive index of water and dilute solutions of sodium chloride for electromagnetic waves of frequency about 10^8 per second. With an exciter of the well-known Hertz type he set up a system of stationary waves along two parallel wires suspended partly in air and partly in a long tank containing water or a salt solution. A comparison of the wave-lengths in the two sections of the wires, determined by a modification of Ruben's bolometer method, gave the results tabulated below where λ is the conductivity of the water or solution, l_a the half wave-length in air, l_ω the half wave-length in water or the solution, and n the index of refraction. The temperature was always between 16.8° and 17.2°.

Substance.	$\lambda \times 10^6$	l_a	l_ω	n	n^2
Water.	7.9	292.4	34.1	8.57	73.5
NaCl solution.	140.0	292.4	33.8	8.65	74.8
"	484.0	292.4	33.0	8.86	78.5

When the conductivity was greater than 532×10^6 he was unable to determine the position of the nodes in the solution owing to the increased damping of the oscillations.

A much more extensive and thorough investigation of the same subject, by a similar method, was published by P. Drude[2] in 1896. He used an oscillator of the Blondlot type and determined the position of the nodes along two parallel wires by the aid of Zehnder's vacuum tube. Great care was taken to develop a method of observation as free as possible from consistent errors and to apply suit-

[1] E. Cohn, Wied. Ann., 45, p. 370, 1892.
[2] P. Drude, Wied. Ann., 59, p. 17, 1896.

able corrections for such errors as could not be eliminated. Containing vessels, for the liquids experimented upon, of various dimensions and materials were tried and their effect on the results determined. After a comprehensive series of observations he was led to the conclusion that, up to a conductivity $\lambda = 5.3 \times 10^{-3}$, the square of the index of refraction of a solution varies less than one per cent. from the value for pure water. Solutions of copper sulphate and sodium chloride were used and the frequency of the electromagnetic waves was about 4×10^8 per second. For higher conductivities he found that n^2 decreased when the concentration was increased and reached a value about 10 per cent. less than that for pure water when λ equalled 40×10^{-3}.

In 1899 W. D. Coolidge,[1] using Drude's method, found the wave-length in water and in a copper sulphate solution of 6.5 times as great conductivity to be the same within 0.2 mm. at a temperature 19.2°. His mean result for the half wave-length in water was 66.184 cm., and in the $CuSO_4$ solution 66.202 cm.

Ernest Lecher,[2] in a footnote to an article on the "Measurement of Dielectric Constants by Means of Hertz Oscillations," states that the capacity of a condenser, measured by his method with Ruhmkorff oscillations, was just the same whether a vessel between its plates contained distilled water or a ten per cent. solution of sulphuric acid. The condenser plates were 28 cm. apart and the vessel containing the liquid was 27 cm. long.

E. B. Rosa[3] in 1891 measured the force between a fixed and a movable electrode by means of a torsion balance and found that it decreased when hydrant water or minute quantities of acid or copper sulphate were added to the distilled water surrounding the electrodes. The difference of potential required for these measurements was furnished by a gravity battery and was mechanically reversed about fifty times per second. He thinks that the observed decrease in force was due to polarization of the electrodes.

The investigations of Drude and Coolidge indicate that the index of refraction of a dilute electrolytic solution, for electromagnetic

[1] W. D. Coolidge, Wied. Ann., 69, p. 134, 1899.
[2] Ernest Lecher, Wied. Ann., 42, p. 152, 1891; Phil. Mag. (5), 31, p. 181, 1891.
[3] E. B. Rosa, Phil. Mag. (5), 31, p. 188, 1891.

waves, is independent of the concentration and the results of Cohn do not appear of sufficient weight to controvert this conclusion. Hence, assuming Maxwell's relation $k = n^2$, we might expect the dielectric constant of such a solution to be the same as that of pure water. But since this relation has been shown to fail in many instances, this deduction cannot be rigorously maintained without corroborative evidence from direct determinations of the dielectric constants of solutions. Published results of such determinations are meagre and inadequate, but they indicate that the change in dielectric constant with concentration is very small in comparison with the corresponding change in conductivity.

After a preliminary experimental and theoretical study of several of the best known direct methods, it was decided that small variations in dielectric constant could be most accurately determined by measuring the force between two electrodes immersed in the solution and maintained in the same relative position. The principal sources of error in such a method may be expected to arise from the polarization of the electrodes, from changes in their relative position, due to convection currents in the liquid or to inaccuracies in adjustment, and from errors in determining their difference in potential. The extent to which these difficulties have been overcome will appear in the following discussion. The first apparatus that gave intelligible results consisted of two electrometers, constructed on the attracted disk type, connected in parallel between the earth and one terminal of the spark-gap of a Hertzian plate-oscillator, having a frequency of about 4×10^7 vibrations per second.

The movable electrode or needle of one of the electrometers was shaped like the longitudinal section of a dumb-bell and suspended at its center of gravity by a fine platinum rhodium wire. The aluminum disks, of which it was made, were 3.8 cm. in diameter and 0.27 mm. thick, and their centers were 5 cm. apart in a horizontal line. The plane of the needle was vertical, and, in the zero position, parallel to and midway between the planes of the two fixed electrodes. These electrodes were slightly greater in area than the disks, and were placed adjacent to opposite ends of the needle, so that when charged they tended to rotate it in the same direction. An inverted glass bell-jar was clamped against the lower surface of the bed-plate

of the instrument and filled nearly to the level of the electrodes with strong sulphuric acid to absorb the moisture from the inclosed air. A small glass rod was suspended from the center of the needle by a platinum wire, and served to damp its vibrations. The other electrometer was constructed in essentially the same way, except that the electrodes were copper cylinders, about 2.9 mm. in diameter, with their axes lying in a horizontal plane and the acid and damper were dispensed with. A funnel passing through a hole in the bed-plate permitted the introduction of the liquid experimented upon, and a stop-cock in the bottom of the bell-jar facilitated its removal.

Both suspensions were provided with concave mirrors so adjusted that, when the needles were in the zero position, the two images of an illuminated wire, placed above the center of a scale some distance in front of the electrometers, were superposed on the middle division of the scale. When the oscillator was put in action both images were deflected from the center in the same direction and after their position became sufficiently steady they were caused to return to the center by suitably turning the torsion heads at the top of the suspensions. The action of the oscillator was then interrupted and the angles of torsion calculated from the resulting scale deflection of the images. When a sufficient number of similar independent observations had been made the solution in the second electrometer was changed and the operation repeated.

Owing to variations in the intensity of the oscillating current and to irregularities in the intervals between the wave trains sent through the electrometers it was at first found quite impossible to bring the two images together on the central division of the scale. The former difficulty was due to the varying condition of the surfaces of the spark balls of the oscillator and was to a great extent eliminated by rotating these balls at a high speed in opposite directions in a plane parallel to the spark gap. This operation apparently produced an effect similar to that of a strong air blast though the gap or of a strong magnetic field perpendicular to it and in the present instance was much more convenient than either of these devices. The latter difficulty was due to irregularities in the action of the interrupter to the induction coil that energized the oscillator. After extensive experiments with various well-known

forms of break, the following construction was found to give by far the most satisfactory results. A german silver wire, 0.8 mm. in diameter and 88 cm. long, was stretched between the poles of a powerful horseshoe magnet, and a platinum mercury contact was attached to its center and very near the magnet. The primary current of the induction coil was passed through this wire in such a direction that it was raised by the magnetic field and the circuit was broken. The tension of the wire could be varied at will through a long range by a guitar string winch, and the relative time of closed and open circuit could be varied by raising or lowering the mercury cup by a screw adjustment. The mercury was covered with alcohol, and a Willyoung adjustable condenser was connected in parallel with the contact. When the instrument was in good condition and properly adjusted no spark was produced at the break, and the interruption could be maintained regular for an indefinite time. This form of break has been used by several investigators, but I am uncertain who first introduced it.

With these appliances the two images could be adjusted to the center of scale within an error of one mm., but they could be held there only a short time. Calling A the observed angle of torsion for the electrometer containing the solution and B that for the other eletrometer the ratio A/B is proportional to the ratio of the forces acting on the needles when in the zero position and any change in this ratio when the solution is changed indicates a corresponding difference in the dielectric constants of the solutions. Distilled water of conductivity about 4×10^{-6} and ten copper sulphate solutions varying in concentration from 5×10^{-5} to 4×10^{-4} gram equivalents per liter were tested in this way. The values for A/B thus obtained were very irregular not only for the different solutions but to a slightly less extent for the same solution. The average values for the very dilute solutions showed a small irregular increase with the conductivity but when the concentration was greater than about 2×10^{-4} they were irregularly distributed about the mean value for distilled water. The temperature was so nearly constant during the series of observations that the irregularities could not be attributed to its variations and it seemed possible that they might be due to changes in the constant of the electrometer, caused by

completely emptying and refilling it between each set of observations, or to corrosion of the copper electrodes. These electrodes were consequently replaced by platinum ones of a similar construction and the solutions were made in the electrometer itself, by adding small quantities of a solution of copper sulphate to the water it contained, thus leaving the suspended system entirely undisturbed. After each addition of the salt sufficient time was allowed to elapse for the solution to become uniform before observations were undertaken. The results were similar to those before obtained and only slightly less irregular. They are graphically represented in Fig. 1

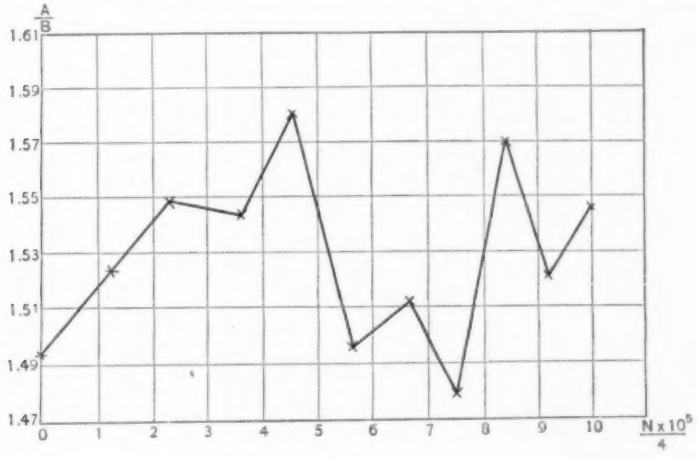


Fig. 1.

where the abscissas are proportional to the concentration N in gram equivalents per liter and the ordinates are proportional to the ratio A/B.

Although these results indicate a slight increase of dielectric constant with the concentration the method appears incapable of giving a trustworthy determination of the relation. This failure is believed to be due to several causes. In the first place the oscillator became less regular in its action as the conductivity of the solutions increased and frequently ceased to act altogether so that some of the observations may have been made with Ruhmkorff in place of Hertzian currents. In this case, on account of the small superficial area of the electrodes, an appreciable polarization may have been

produced. After some of the settings a slow drift of the image, from the electrometer containing the solution, was noticed that may have been due to the subsidence of such a polarization. At all events the direction of the drift was harmonious with such an explanation. Small convection currents in the air electrometer or slight brush discharges from the comparatively sharp edges of the needle may have introduced irregular errors. Furthermore the fixed and movable electrodes of this instrument were placed very near together in order to make its capacity nearly equal to that of the other electrometer and it is obvious that small errors in the position of the needle would produce relatively large errors in the observed force. Since the difficulty experienced in setting the needle in the zero position increased with the conductivity of the solution this source of error probably accounts for a large part of the observed irregularity. It is apparent that errors of setting would have had less effect on the result if electrometers of the quadrant type had been used in place of those of the attracted disk type. The latter type was adopted because I had been unable to devise a high frequency oscillator that would produce wave trains of sufficient energy to cause readable deflections with the quadrant form and with low frequencies polarization completely masked the results. A slight change in one of the oscillators already tried, without success, obviated this difficulty and after some further preliminary work the disposition of apparatus about to be described was adopted and found to give satisfactory results.

Fig. 2 is a diagram of the oscillating system and electrometer

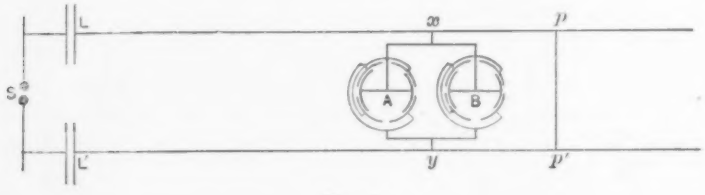


Fig. 2.

connections. Two large Leyden jars L, L' were placed on glass insulating stands and their outside coatings connected to an adjustable spark gap S. The parallel parts of these connections were brass rods, about 12 mm. in diameter, 20 cm. long and 10 cm.

apart, fitted with right angle clamps at their free ends that supported shorter rods 6 mm. in diameter and terminating in brass balls 2.3 cm. in diameter. The inside coatings of the jars were connected with two parallel copper wires, about eight meters long and 20 cm. apart, that extended in an inclined plane from the oscillator to the ceiling of the room. One pair of quadrants, the needle and the brass case of each of the electrometers A and B were connected together and to a point x on one of the parallel wires about 2.5 meters from L. The remaining quadrants were insulated from the case and connected to the point y on the other wire equidistant from L'. A bridge pp' was arranged to slide freely on the wires and its motion was controlled by a flexible cord passing over pulleys in the ceiling of the room and extending to the left hand end of the observing scale. When it was in contact with xy the observed electrometer deflections were very small, but as it was drawn away they gradually increased and finally exceeded the length of the scale. The frequency of electrical oscillation of the system was about 10^6 vibrations per second and it was not appreciably affected by the changes in position of the bridge necessary to produce any desired deflection within the limits of the scale. When first set up the inside coatings of the jars were connected to the spark gap and the outside coatings to the parallel wires, but with this arrangement the energy of the oscillations was so small or the damping so great that no deflection could be obtained when the conductivity of the liquid in either electrometer was greater than that of pure water. With the reverse connection appreciable deflections were produced in liquids of conductivity as high as that of a decinormal solution of copper sulphate. The cause of this great difference in action is not apparent, but it may be due to differences in effective insulation.

The two electrometers A and B were made as near alike as possible and each was filled with water, or a solution, so that their electrostatic capacities were nearly equal. The connecting wires for corresponding parts were of equal length and diameter and were so arranged that the two branches of the circuit had nearly the same self-induction. Each electrometer was supported by a brass bed plate, 15 cm. in diameter and 5 mm. thick, rigidly attached, by means of three leveling screws, to a substantial wooden frame that

was held in position on a brick pier by a heavy weight. A brass tube 2 cm. in diameter and 30 cm. long was supported over a hole in the center of the plate by a flange and shoulder, and carried at its upper end a torsion head from which the needle was suspended by a platinum rhodium wire about 20 cm. long and 0.05 mm. in diameter. A small rectangular window near the base of this tube was closed by plate glass and served to admit light to a small concave mirror on the suspended system. The fixed electrodes were in the form of shortened quadrants of a cylindrical surface 9 cm. in diameter and 3 cm. high and were supported by brass shanks 15. mm. in diameter that passed up through hard rubber plugs screwed into holes in the bed plate. The insulating spaces between their adjacent edges were about 4 mm. wide. The needle was bent to coincide with two opposite quadrants of a concentric cylindrical surface 7 cm. in diameter and 2 cm. high and the shank by which it was supported carried a small binding screw at its upper extremity by which it was clamped to the lower end of a short brass wire soldered to the suspending wire. Both cylindrical surfaces were normal to the bed plate and their axes coincided with the suspension. The needle and electrodes were cut from sheet German silver 0.2 mm. thick and were thoroughly plated with platinum and platinized before being bent into shape. They were so constructed that all soldered connections came above the surface of the liquid in the electrometer. A glass bell-jar, about 11 cm. in diameter and 700 cc. capacity, with a stopcock in its closed end was clamped in an inverted position against the under surface of the bed plate and surrounded by a cylindrical copper vessel that served to shield the electrodes from external electrostatic influences and to convey a current of water from the city mains for the purpose of regulating the temperature when necessary. Generally, however, the temperature of the room was more constant than that of the water and the latter device was seldom used. The temperature was indicated by a mercury thermometer supported by the bed plate with its bulb very near the fixed electrodes and as far as possible from the needle.

The suspensions of the two electrometers were 30.5 cm. apart and the line joining them was parallel to and 177.8 cm. in front of an observing scale, 150 cm. long, supported by a wooden stand on

a brick pier and so placed laterally that a normal erected to its central division would pass midway between the electrometers. Directly above this normal a fine steel wire was stretched vertically across a small window in the supporting stand and illuminated by an incandescent electric lamp and a condensing lens. The concave mirrors on the suspensions were so adjusted that the two images of this wire were superposed on the central scale division when the electrometer needles were symmetrically situated with respect to the fixed electrodes. The images were very sharp and slightly less in width than the division marks of the scale, so that readings could be easily made to fractions of 1 mm. when desirable. The motion of each torsion head was controlled by an endless belt of fine steel wire that engaged a wooden lever arm attached to the head and, after passing over a system of fixed pulleys, circulated several times about a small wooden reel attached to the scale support. The belts were kept taut by brass spiral springs and worked so smoothly when the reels were turned that no tremor of the images were observed during the motion. The stretched wire interrupter described above, the condenser, and the switch for the primary circuit of the induction coil were placed within easy reach of the observer at the scale. The electrometers and scale were enclosed in a tent of dark cloth supported by a wooden frame attached to the floor of the room and mechanically insulated from the brick piers.

The relative conductivity of the solutions used was determined by Kohlrausch's telephone method with the aid of two pairs of small platinum test electrodes similar to the "Tauchelektroden" for poorly conducting liquids, described by Kohlrausch and Holborn.[1] These electrodes were supported by the bed plates of the electrometers, in a position diametrically opposite to the thermometers, and were insulated from them by glass tubes covered with paraffin. They were kept in a fixed position throughout the investigation and could be insulated or connected to either arm of the resistance bridge by suitably placing copper connectors in a mercury commutator constructed from a large block of paraffin.

After carefully cleaning the electrodes and bell-jars the electrometers were assembled and filled with distilled water until its sur-

[1] Kohlrausch and Holborn, "Leitvermögen der Elektrolyte," edition of 1898, p. 18.

face was about one cm. above the top edges of the electrodes and 3 cm. below the bed plate. Determinations of the conductivity of the water showed that it increased about 30 per cent. per day and this was judged to be due to imperfect removal of the salts of the plating bath from the surface of the platinized electrodes by the extended washing in hot and cold water to which they had been subjected. Consequently the electrometers were allowed to remain undisturbed about one week and were then refilled with fresh distilled water. This operation was repeated at nearly equal intervals for three months and measurements then show that the rate of increase in conductivity had been greatly reduced. Nearly equal volumes of all the solutions investigated were used so that the free surface always occupied the same position and produced the same effect on the measurements.

The method of operation in taking the observations was as follows: The image of the illuminated wire due to one of the electrometers (say B) was set at a convenient position on the scale. The interrupter was started and when its pitch became regular the bridge pp' was moved along the parallel wires till the image B came to the central scale division (75th in this instance), keeping B stationary by slight back and forward motions of the bridge, corresponding to the irregularities in the intensity of the oscillating current, the image from electrometer A was brought into coincidence with it by turning the proper torsion head. This adjustment could be easily made to a small fraction of a millimeter and maintained constant for an indefinite time by slight motions of the bridge. The primary circuit of the induction coil was then broken and after the images had come to rest their position on the scale was recorded. Observations of the temperature and conductivity of the solutions in the electrometers were made and recorded. From six to ten sets of observations were taken in this manner, starting each time with the image from B in a different position, in order to obviate prejudice in making the settings. About 2 cc. of a centinormal solution of copper sulphate were then added to the water in electrometer A and the apparatus allowed to remain undisturbed for about twenty-four hours in order that the salt might become uniformly distributed. On succeeding days similar series of observations were made and the concentration

of the solution was increased by successive additions of from 2 cc. to 4 cc. of centinormal or decinormal solutions of copper sulphate until six different solutions had been tested. The solution in A was was then kept constant and eight different copper sulphate solutions were formed in B and tested in a similar manner. After this series of observations B was emptied, thoroughly rinsed out and refilled with fresh distilled water. Similar sets of observations were made on this water and on five different potassium chloride solutions formed as described above. It will be noticed that in the first series of observations the solutions in A were compared with water in B, while in the last two series the comparison was between various solutions in B and a constant solution in A. This procedure was adopted since it was found that the conductivity of the dilute solution in A was much more constant than that of the water.

When the two images were superposed on the 75th scale division the needles of the electrometers were symmetrically situated with respect to the fixed electrodes and the electrostatic forces acting on them were balanced by the torsion of the suspending wires. Since the displacements were always small the torsional couples produced by the wires were proportional to the angles through which they were twisted and when the current was turned off the images were deflected along the scale a distance depending on the magnitude of the torsion. The formulæ ordinarily used in reducing scale readings to angular measure are obviously inapplicable when the apparatus is arranged as above described since the normals to the minors are not perpendicular to the scale when the images are superposed on the 75th division. It is, however, easy to show that the angle of torsion δ_a corresponding to a deflection of the image from electrometer A amounting to x_a scale divisions is given by the expression

$$\delta_a = \tfrac{1}{2} \tan^{-1} \frac{S}{1 + T\frac{x_a}{A^2}} \cdot \frac{x_a}{A^2}$$

where S is the normal distance between the scale and the line joining the electrometer suspensions, A the distance from the minor to the 75th scale division, T one half the distance between the suspen-

sions, and x_a is measured on the opposite side of the normal to the 75th division to that on which the electrometer is situated. Similarly for a deflection x_b on the same side of this normal as electrometer B.

$$\delta_\beta = \tfrac{1}{2}\tan^{-1}\frac{S}{1 - T\frac{x_b}{A^2}}\cdot\frac{x_b}{A^2}.$$

Expanding these expressions in ascending powers of x/A^2

$$\delta_a = \frac{S}{2A^2}x_a - \frac{ST}{2A^4}x_a^2 - \frac{S(S^2-3T^2)}{6A^6}x_a^3 + \cdots$$

$$\delta_\beta = \frac{S}{2A^2}x_b + \frac{ST}{2A^4}x_b^2 - \frac{S(S^2-3T^2)}{6A^6}x_b^3 + \cdots$$

From the dimensions of the apparatus it follows that $S = 177.8$ cm., $T = 15.25$ cm. and $A = 178.5$ cm. Substituting these values

$$\delta_a \times 10^4 = 27.9\, x_a - .0134\, x_a^2 - .000283\, x_a^3$$

$$\delta_\beta \times 10^4 = 27.9\, x_b + .0134\, x_b^2 - .000283\, x_b^3.$$

A table of the values of $\frac{1}{2}\delta_a \times 10^4$ and $\frac{1}{2}\delta_\beta \times 10^4$ was calculated from these formulæ for values of x_a and x_b varying by one or two centimeters from zero to 60 cm. and the results were plotted to form curves with ordinates proportional to the angles and abscissas proportional to the deflections. The reduction of the observations was made with the aid of these curves and a series of calculations for points well distributed along the curves showed that the results obtained by this method never differed by so much as one tenth of one per cent. from the results calculated directly from the formulæ.

Since the two electrometers were symmetrically connected to the same points of the parallel wires they were acted upon by equal differences of potential when the oscillator was in action. Hence, if δ_a and δ_β represent the observed angles of torsion when the dielectric constants of the contained liquids are κ_a and κ_β respectively it is obvious that

$$\frac{\delta_a}{\delta_\beta} = \frac{C_a\kappa_a}{C_\beta\kappa_\beta} \qquad (1)$$

where C_α and C_β are constants depending on the dimensions of the electrometers and the elastic properties of the suspending wires. Similarly for angles δ_α' and δ_β' corresponding to liquids of dielectric constant κ_α' and κ_β',

$$\frac{\delta_\alpha'}{\delta_\beta'} = \frac{C_\alpha \kappa_\alpha'}{C_\beta \kappa_\beta'}. \tag{2}$$

Hence dividing (2) by (1)

$$\frac{\delta_\alpha'}{\delta_\beta'} \cdot \frac{\delta_\beta}{\delta_\alpha} = \frac{\kappa_\alpha'}{\kappa_\alpha} \cdot \frac{\kappa_\beta}{\kappa_\beta'} \tag{3}$$

from which either ratio $\kappa_\alpha'/\kappa_\alpha$ or $\kappa_\beta/\kappa_\beta'$ can be determined when the other is known.

Owing to slight differences in the temperature of the liquids experimented upon it become necessary to reduce the observations to some standard temperature and the necessary corrections were calculated in the following manner. Write

$$\kappa_\alpha = \kappa(1 + \beta_\alpha t_\alpha)$$

where κ_α is the value of κ when the liquid is at temperature t_α and

$$\beta_\alpha = \frac{1}{\kappa} \frac{\partial \kappa}{\partial t}.$$

Similar expressions for the other κ's may be written

$$\begin{aligned} \kappa_\alpha' &= \kappa'(1 + \beta_\alpha' t_\alpha') \\ \kappa_\beta &= \kappa_0(1 + \beta_\beta t_\beta) \\ \kappa_\beta' &= \kappa_0'(1 + \beta_\beta' t_\beta'). \end{aligned}$$

Since the temperature differences were small and the liquids used were all water or moderately dilute solutions, the temperature coefficients β_α, β_α', etc., were assumed to be the same and equal to that for pure water at the mean temperature of the observations. Granting this assumption

$$\frac{\kappa_\alpha}{\kappa_\beta} = \frac{\kappa}{\kappa_0}\left\{1 + \beta(t_\alpha - t_\beta)\right\}$$

and

$$\frac{\kappa_\alpha'}{\kappa_\beta'} = \frac{\kappa'}{\kappa_0'}\{1 + \beta(t_\alpha' - t_\beta')\}.$$

Hence (3) becomes

$$\frac{\delta_\alpha'}{\delta_\beta'} \cdot \frac{\delta_\beta}{\delta_\alpha} = \frac{\kappa'}{\kappa} \cdot \frac{\kappa_0}{\kappa_0'} \cdot \frac{1 + \beta(t_\alpha' - t_\beta')}{1 + \beta(t_\alpha - t_\beta)}. \tag{4}$$

When B was used as the comparison electrometer the liquid in it was left undisturbed and if we neglect the possibility of small changes in its dielectric constant due to a slight increase in conductivity we have as a first approximation $\kappa_0 = \kappa_0'$ and

$$\frac{\kappa'}{\kappa} = \frac{\delta_\alpha'}{\delta_\beta'} \cdot \frac{\delta_\beta}{\delta_\alpha} \cdot \frac{1 + \beta(t_\alpha - t_\beta)}{1 + \beta(t_\alpha' - t_\beta')} \tag{5}$$

where κ' is the dielectric constant of the solution experimented upon and κ that of the water from which it was made. β was taken equal to $-.004$ which is very nearly the mean of the best published determinations of this constant for temperature changes similar to those observed. When the functions of the two electrometers are reversed and A is used as comparison electrometer the reduction formula is the same as (5) with the α's and β's interchanged.

After the first series of observations had been made and reduced it was found that the value of the ratio $\delta_\alpha/\delta_\beta$ depended on the manner in which the test electrodes were connected to the resistance bridge. This effect was undoubtedly due to currents induced in the connecting wires by the oscillating current in the electrometer circuit and the following observations show that it was constant so long as the connections were unchanged. Three sets of connections were used, P_1, test electrodes in A and B connected to adjacent bridge arms; P_2, electrodes in A and a rheostat similarly connected, and P_3, electrodes in B and a rheostat similarly connected. When both electrometers were filled with water at temperature 25.°7 the observations lead to the values of $\delta_\alpha/\delta_\beta$ given in Table I.

TABLE I.

Connection.	P_1	P_3
$\frac{\delta_\alpha}{\delta_\beta}$	1.045	1.031
	1.044	1.033
	1.047	1.037
	1.048	1.035
	1.045	1.033
Mean	1.046	1.034

The scale deflections from which these results were calculated range from $x_a = 29.1$ cm. to $x_a = 56.9$ cm. and from $x_b = 27.4$ cm.

to $x_b = 51.5$ cm. Unfortunately this set of observations did not involve the connection P_2 and the corresponding value of $\delta_\alpha/\delta_\beta$ could only be obtained by calculation from the mean value of $\delta_\alpha'/\delta_\beta'$ corresponding to a solution for which the ratio κ'/κ had been determined from the observations with connections P_1 and P_3. The solution for which the errors of observation were smallest was adopted for this purpose and the result thus obtained was $\delta_\alpha/\delta_\beta = 1.05$ for connection P_2. Table II. exhibits the observations and calculated results for a solution of copper sulphate of conductivity 3.86 times that of the water from which it was made. The letters in the column headed C indicate the connection of the test electrodes that existed when the corresponding scale deflections x_a and x_b were observed and the given values of κ'/κ were calculated from $\delta_\alpha'/\delta_\beta'$ with the acid of the value of $\delta_\alpha/\delta_\beta$ for corresponding

TABLE II.

x_a	x_b	$\delta_\alpha'/\delta\beta'$	C	κ'/κ
56.3	51.2	1.037	P_2	.987
53.7	49.6	1.024	P_3	.990
51.6	46.6	1.050	P_2	1.000
47.2	43.6	1 033	P_3	.999
44.8	40.7	1.054	P_2	1.002
40.7	37.8	1.034	P_3	1.000
39.3	35.9	1.050	P_2	1.000
35.8	33.3	1.036	P_3	1.003
34 1	31.3	1.049	P_2	.999
39.5	36.8	1.033	P_3	.999

connections. The mean error of the individual values of κ'/κ, calculated from the average value in the usual manner, is .0052 and since this is somewhat greater than the average of the mean errors for the other sets of observations, the results given in the table convey a fair idea of the general accuracy of the observations and reductions.

During the last two series of observations, A was used as comparison electrometer, and contained a dilute solution of copper sulphate. The test electrodes were always insulated while the electrometer settings were made, and hence but one value of $\delta_\alpha/\delta_\beta$ was obtained when B was filled with distilled water. More or less ex-

tensive sets of observations on various solutions of copper sulphate and potassium chloride lead to the results given in Table III. The

TABLE III.

D	S	t_1	t_2	λ_1	λ_2	κ'/κ	E	N
2	$CuSO_4$	26.0	26.0	1.11	1.69	.998	.0034	8
3	"	26.3	26.3	1.40	2.92	1.010	.0037	8
4	"	25.9	26.0	1.61	3.86	.998	.0052	10
5	"	25.2	25.3	1.74	4.68	1.003	.0028	10
6	"	24.3	24.3	1.73	4.64	1.002	.0053	6
8	"	23.9	23.9	2.00	5.63	1.009	.0021	9
9	$CuSO_4$	23.8	23.8	1.04	2.95	1.000	.0024	10
10	"	22.9	22.9	1.04	3.62	1.004	.0023	10
11	"	23.1	23.1	1.12	4.29	1.006	.0028	8
12	"	23.0	23.1	1.16	7.07	1.003	.0052	8
13	"	22.1	22.5	1.15	10.62	1.002	.0046	7
15	"	23.6	24.0	1.27	18.45	.986	.0177	4
16	"	24.9	25.5	1.28	19.74	.994	.0059	7
17	"	25.2	25.9	1.30	32.45	1.007	.0180	6
19	KCl	25.3	25.3	1.10	3.87	.997	.0047	8
20	"	24.8	25.0	1.15	7.65	.986	.0023	6
22	"	25.1	25.7	1.25	11.68	.990	.0056	6
23	"	25.3	25.8	1.27	14.70	.989	.0046	6
24	"	25.5	25.9	1.31	20.79	.989	.0141	6

independent series of observations are separated by horizontal spaces, and the days of July, 1901, on which the observations were made are indicated in column D. The salts in the solutions examined appear under S. t_1 is the temperature of the solution in the comparison electrometer, and t_2 that of the solution in the test electrometer. λ_1 is the conductivity of the former solution relatively to its value when the ratio δ_a/δ_β was determined, and λ_2 is the conductivity of the latter solution relatively to that of the water from which it was made. κ'/κ is the mean of N independent determinations of the ratio between the dielectric constants of the latter solution and distilled water, and E is the mean error of the individual values of κ'/κ These results were derived from the observations by equation (5), on the assumption that the dielectric constant of the liquid in the comparison electrometer remained constant, except for temperature changes, during the several series of observations. Hence, if κ'/κ

had been found to vary with λ_2 corrections would have been necssary on account of the observed variationin λ_1. But, since λ_2 varied much more rapidly than λ_1 and the variations of κ'/κ from unity were of the same order of magnitude as the errors of observation, this correction becomes unnecessary. The uniformly low values obtained in the third series are probably due to an error of about one per cent. in the determination of the ratio $\delta_\alpha/\delta_\beta$ when the test electrometer contained distilled water.

The conductivity of the distilled water from which the solutions were made, expressed as the reciprocal of the resistance in ohms of a column one centimeter long and one square centimeter in section, was about 7×10^{-6}. Consequently the maximum conductivity of the solutions tested was not over 2.5×10^{-4} which, for the copper sulphate solutions, corresponds to a concentration of about 0.003 gram equivalents per liter.

Considering the low conductivity of the solutions, the large area of the electrodes, and the high frequency of the oscillating current, the polarization might be expected to be vanishingly small and in fact the observations gave no evidence of such a disturbance. The electrometer needles were nearly dead beat and the images from the mirrors on the suspensions always came to rest in fifteen or twenty seconds without the least sign of drift due to subsiding polarization. During the adjustment of the images to the central scale division no gradual variation of the forces acting on the needles such as would be produced by accumulating polarization was observed. On account of irregularities in the action of the oscillator some of the settings occupied much more time than others and hence offered an opportunity for polarization to produce an increased effect, but the results calculated from these observations never differed from those occupying less time by more than the ordinary errors of observation.

When the images had come to rest after the current was turned off they remained perfectly steady so long as the apparatus was left undisturbed. Consequently the convection currents in the liquids must have been inappreciable.

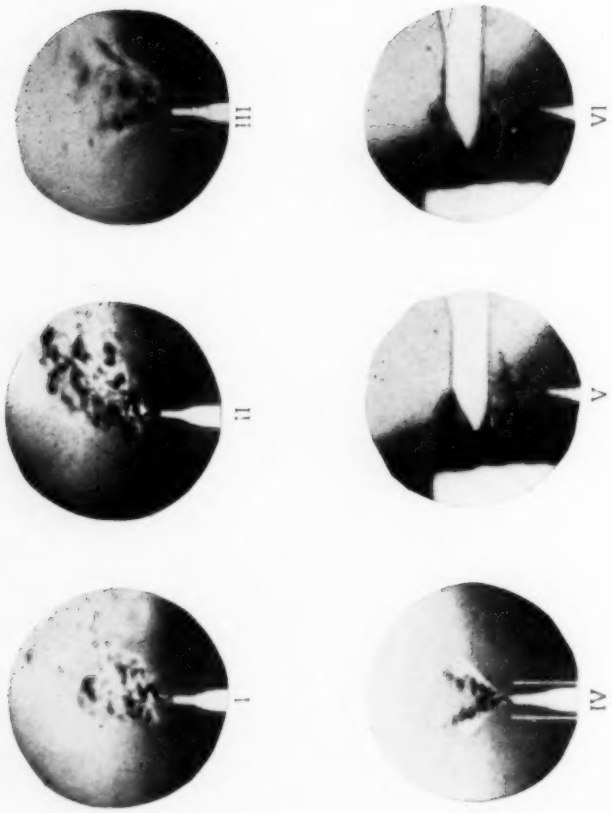
In order to determine whether the deflections were influenced by changes in the torsional rigidity of the suspending wires due to the

heating effect of the currents they transmitted the period of torsional vibration of the needle of electrometer A was determined by comparison with the pendulum of the laboratory clock with the aid of a double chronographic record. The mean of a large series of observations gave 5,965 seconds when no current was passing through the suspension and 5,967 seconds when it was conducting a current larger than that used during any of the electrometer settings. Since the difference between these values lies within the probable errors of observation it is evident that the heating effect was negligible.

An inspection of the values of κ'/κ given in Table III., is sufficient to show that they do not differ from unity by greater amounts than might have been expected if distilled water had been used in the electrometers throughout all the observations in place of the solutions actually used. Hence it is highly probable that the dielectric constant of dilute electrolytic solutions, of conductivity less than 2.5×10^{-4}, is the same as that of pure water in electric fields varying at the rate of 10^6 oscillations per second.

It is obvious that the degree of electrolytic dissociation of such solutions is very large and the observed lack of change in dielectric constant may be due to this fact but the point cannot be definitely decided until determinations have been made with much more concentrated solutions. Some such observations have been made but they are not sufficiently accurate to lead to definite results and I shall therefore defer the discussion of them until I have had time to carry out a more thorough investigation of the subject with an apparatus modified to render it more adaptable to measurements with highly conducting liquids.

PHYSICAL LABORATORY, BROWN UNIVERSITY,
PROVIDENCE, R. I., Nov. 30, 1901.



TUFTS: ACTION OF SOUND WAVES ON UNIGNITED GAS JETS.

A STUDY OF THE ACTION OF SOUND WAVES ON UNIGNITED JETS OF GAS.

By F. L. Tufts.

IN this work the Schlerin method of investigation, used and described by Toepler, C. V. Boys, R. W. Wood and others, was applied to the study of the behavior of unignited jets of gas when subjected to the action of sound waves. Photographs were taken, giving a number of instantaneous positions of a jet of unignited illuminating gas when acted on by the sound waves from an ordinary organ pipe. The orifice from which the gas issued was rectangular in shape and was 4.0 mm. by 0.5 mm. in cross section. It was placed about 6 cm. from the open end of an organ pipe which spoke a fundamental note of 100 vibrations per second. A thin rubber diaphragm was placed across the inside of the pipe at the node of the fundamental note, and thus protected the jet from the action of direct currents of air while it did not interfere with the speaking of the pipe. The jet tube was placed so as to expose the broad side of the gas stream to the to-and-fro motion of the air particles in the sound wave.

The photographs taken showed the sinuous shape of the jet near the orifice, the fan-shaped spreading of the gas particles as they recede from the orifice and, for low velocities of efflux, the division of the jet into two.

Plates I. to IV. are reproductions of some of the photographs taken. Plates I. and II. show two successive positions of the gas stream. In Plates III. to IV. the velocity of efflux was less and the tendency of the stream to divide into two is evident. This is particularly prominent in Plate IV. The sinuous structure of the stream near the orifice is noticeable in all of the photographs, and in most of them traces of it can be found even in the diffused upper portions. The disturbed condition shown in the upper portions of

Plates I. and II. is probably due to air currents in the room. The two pins, shown at the sides of the jet tube in Plates III. and IV., are just one centimeter apart and serve to give an idea of the actual magnitude of the phenomena.

A simple geometrical construction, shown in Fig. 1, gives positions for the gas stream which agree well with the actual positions shown in the photographs. The assumptions upon which the construction is based are the following:

1. Each particle of gas as it issues from the tube moves forward in a straight line, and with a velocity which is the resultant of the combination of the velocity of efflux of the particle and the instantaneous value of the harmonically varying velocity of the air particles about the jet at the instant the gas particle issues from the orifice.

2. This resultant velocity decreases in value as the gas particle recedes from the orifice.

In Fig. 1 the ordinate, *jd*, represents the velocity of eflux of the gas particles. The abscissæ, *dg*, *df*, *de*, *dc*, etc., represent suc-

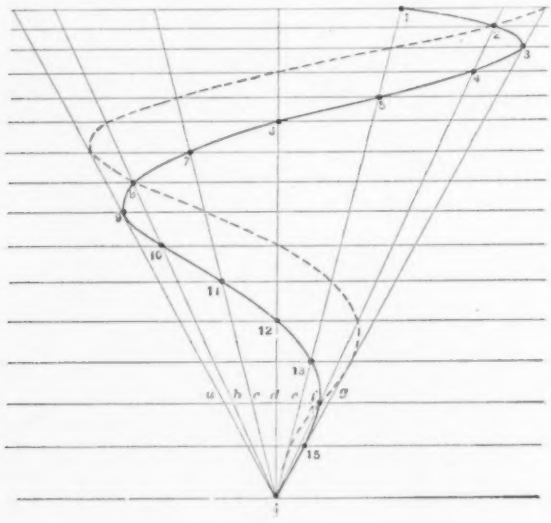


Fig. 1.

cessive values of the harmonically varying velocity of the air particles, due to the sound wave. The lines *jg*, *jf*, *je*, etc., represent the resultant velocities of successive gas particles as they issue from

the orifice. The decrease in the velocity of the gas particles on receding from the tube is taken account of in the spacing of the horizontal lines. The lengths of a radial line intercepted between successive horizontal lines are assumed to be proportional to the velocities with which the gas particle moves as it recedes from the tube.

Suppose the organ pipe used gives a fundamental note of 100 complete vibrations per second, and it is desired to find the positions of successive gas particles following one another from the orifice at intervals of 1/1200 of a second. An inspection of the figure will show that if the distance traveled by a given particle during 1/1200 of a second after leaving the orifice is represented by the line, *j* 15, then the particles which preceded this one at intervals of 1/1200 of a second will occupy the positions marked 14, 13, 12, 11, etc. A curved line connecting these points represents the position of the gas stream at a given instant. Two twelve-hundredths of a second later the position of the gas stream will be that of the dotted line in the figure. The curves thus obtained are very similar to those actually observed in the photographs.

This construction also explains the apparent division of the gas stream into two when the velocity of efflux is small. A low velocity of efflux would be represented by a closer spacing of the horizontal lines in the figure. Under such conditions it is evident that the gas particles would be crowded closer together along the radial lines *jf*, and *jb*, than along any other radial lines and would thus give the gas stream the appearance of dividing into two. The photographs which show a division of the gas stream, show traces in each jet of a structure like that at 2, 3, 4, in the figure.

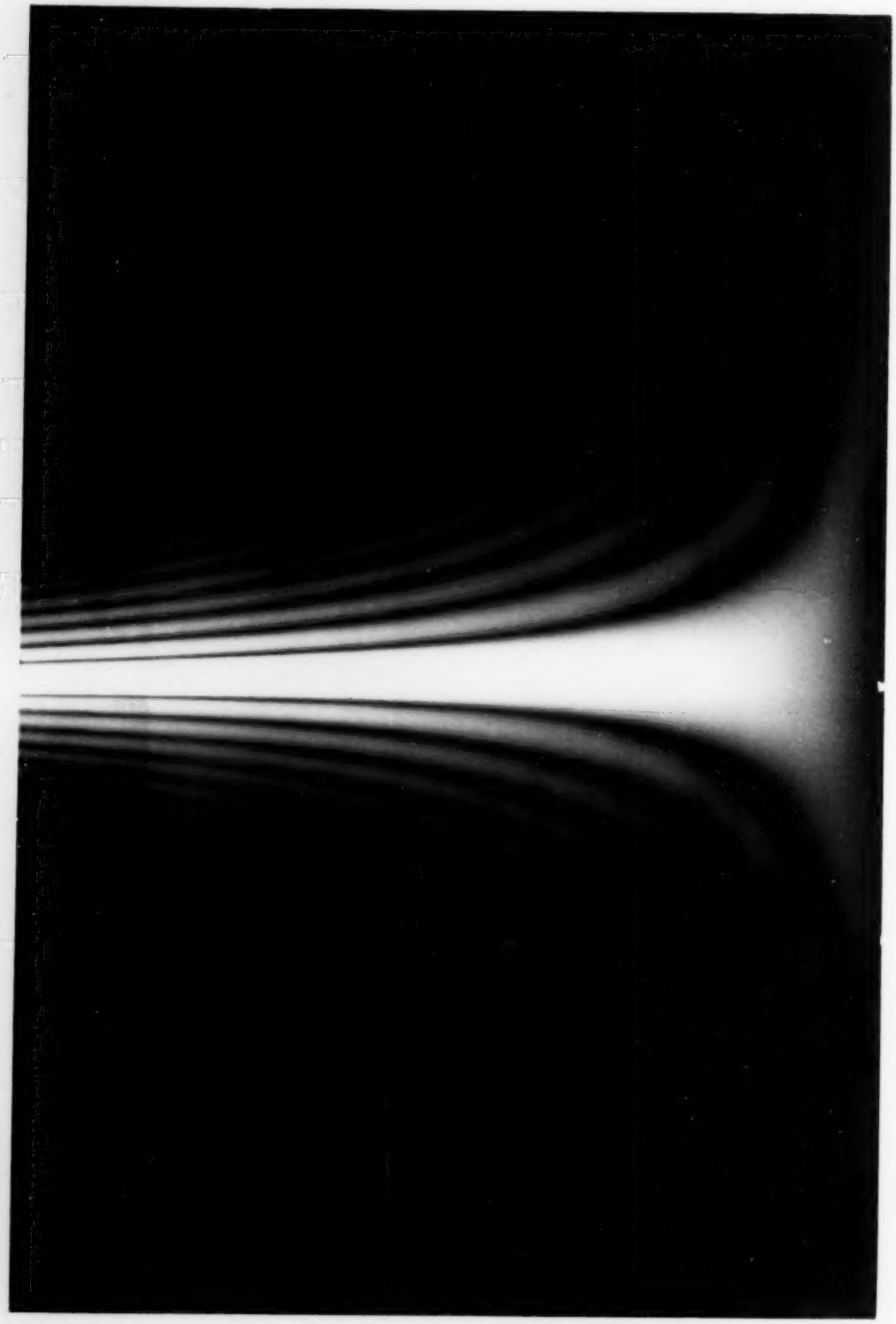
If the source of sound contains over-tones which are at all comparable in intensity to the fundamental, a construction similar to that of Fig. 1, will account for the division of the gas stream into three or more jets, a phenomenon which has been noticed.

In a number of cases the velocity of efflux of the gas from the orifice was measured by means of a gas meter, the pitch of the organ pipe was determined, and the angle *ajg*, Fig. 1, was measured on the photograph of the gas stream. From these data calculations of the amplitude of vibration of the air particles in the sound

wave were made, based on the geometrical construction given above. The results agreed very well with those obtained by other methods.

The Schlerin method was also employed in studying the vibrations of the tongue of air in the mouth of a speaking organ pipe. A stopped flute pipe of rectangular cross section was used. The two sides of the pipe were of plate glass and the air with which the pipe was blown passed over a surface of ether before entering the pipe, thus changing its optical density and making it readily visible in the Schlerin apparatus. A number of instantaneous photographs were taken showing different positions of the tongue of air in the mouth of the pipe. The direction of the vibration within the pipe was obtained by introducing a small jet of unignited illuminating gas in the pipe near its mouth and photographing it on the same plate with the tongue of air.

Plates V. and VI. are copies of two of the photographs showing the tongue of air in the two extreme positions of its swing. The only portions of the pipe shown in the photograph are the lip and a part of the butt containing the air slot. The curve of the gas stream serves to indicate the direction of the vibration within the pipe.



FRANKLIN: DIFFRACTION PHOTOGRAPHS.

SOME DIFFRACTION PHOTOGRAPHS.

By W S. Franklin.

THE accompanying plate is reproduced from one of a number of photographs of shadows cast upon photographic plates from a monochromatic point-source of light distant about sixteen meters from the plate. The objects casting the shadows were about three meters from the plate, and the photographic plate was in each case at right angles to the incident light.

Diffraction photographs were made in this way in the shadows of a number of different objects, with exposures ranging from twenty seconds to seven hours. Among the photographs obtained may be mentioned (1) the shadow of a broad straight-edged plate; (2) the shadow of a steel rod; (3) the shadow of five circular discs. These discs were suspended upon a network of fine wires, each 0.004 cm. in diameter. The shadows of the wires show in the photograph; (4) the shadow of a brass plate with several circular holes bored through it. These photographs, while well suited for lantern projection, are not sufficiently strong for reproduction.

Photograph No. 5, which is reproduced in the accompanying plate, is the shadow of a slit between very carefully worked straight edges of brass. The slit was about 15 centimeters in length, tapering from a width of 0.05 centimeter at the end *a*, to zero width at the end *p*. The original negative was 20×25 cm.

Lehigh University, South Bethlehem, Pa., April, 1901.

NOTE.

Rudolph Koenig.—This well-known authority in the science of acoustics died in Paris on the second of October, 1901. Koenig was the son of a teacher in mathematics at Koenigsberg, in which city he was born on the 26th of November, 1832. At the age of twenty he went to Paris and learned the trade of musical instrument maker under the celebrated maker of violins, Vuillaume. In 1859 he set up for himself as a manufacturer of acoustical instruments.

At the London exhibition of 1862 Koenig first made public his beautiful apparatus for the study of sound waves by means of the manometric flame. The applications of this method to various problems in the analysis of complex tones were subsequently published in three papers in Poggendorff's Annalen. The apparatus constructed in his workshop at Paris was largely of his own design and the experiments which he himself performed with it, and which he described in a series of papers, chiefly in the *Annalen* and in the *Comptes Rendus* of the French Academy, are among the most important contributions to the science of experimental acoustics. The results of his work in this field were in 1882 brought together in his well-known volume entitled "Quelques Expériences D'Acoustique."

Among the best known of Koenig's investigations may be mentioned his study of the fixed tones characteristic of the vowels (1870); his various papers on the synthesis of musical tones; his researches on the theory of the nature of summation and difference tones and his studies of the movement of air in organ pipes. Koenig was fortunate in being able to bring to bear upon his truly scientific work training of two sorts. His knowledge of acoustical theory enabled him to see definitely the nature of the problems demanding solution and his skill as a mechanician made it possible to produce apparatus perfectly adapted to the experimental solution of these problems. To his chosen field he devoted himself absolutely throughout his long career and even during the later part of the nineteenth century, when the interest of physicists was directed so strongly to electricity that the science of sound seemed likely to be altogether submerged and lost from sight, he continued to devote himself to the work which he had elected to do. Koenig's last investigation, completed in spite of failing health and of the growing disabilities of increasing age, was on the "Highest Audible Tones and upon the Inaudible Tones up to 180,000 Vibrations per Second." To the accurate adjust-

ment and tuning of forks and other instruments far beyond the limits of audibility, by means of their difference tones and by the observation of dust figures, he brought to bear the skill resulting from long years of practical experience, during which he had with his own hand made the final adjustment of every instrument constructed in his establishment.

Koenig died in his apartments over the workshops on the Quai d'Anjou, which had for many years been the scene of his scientific activity.

E. L. N.

NEW BOOKS.

Advanced Exercises in Practical Physics. By ARTHUR SCHUSTER, Ph.D., F.R.S., and CHARLES H. LEES, D.Sc. The Macmillan Co. $2.00. Pp. 368.

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In general the methods used are accurate and thorough. One of the strongest features of the book is the careful explanation of errors of observation. The student is frequently shown the effects of errors of observation on a derived result, and his attention is continually directed towards measuring accurately those quantities whose errors would exert the greatest effect. Too many students, perhaps through lack of proper instruction, fail to form any idea of how accurate their result should be, even when they know how accurately each quantity has been measured.

Comparatively few errors are noticed. The first sentence in the paragraph beginning "The fractional error * * *" on page 6 needs a slight revision. On page 14 we find the authors saying, "the two first and the two last observations." The work on mechanics is so good that one is surprised in not finding more on the pendulum. Not a single experiment is given for determining g.

Some manuals seemed designed to teach the principles of Physics, while others are designed to teach laboratory methods. The present work is a happy combination of the two. However, a course based on this book should by all means be supplemented by lectures, problems, and quizzes. In the rapid development of laboratory work there has been a tendency to carry it too far. We find that in some of our schools and colleges practically all of the student's time that is devoted to Physics is spent in experimental work. In doing this the theoretical side is almost entirely neglected. The student learns manipulations, but does he learn Physics? Few students can get a working knowledge, for example, of Ohm's law and its various applications simply from the routine laboratory work and the instructions found in any manual. Laboratory courses should always be supplemented by other work.

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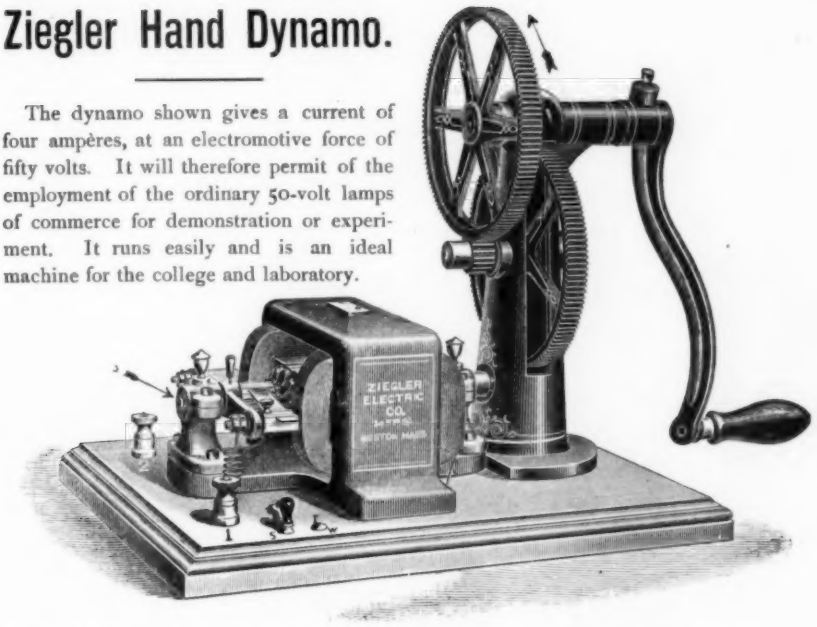

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Contents of Recent Numbers

MAY, 1901.

Henry Augustus Rowland JOSEPH S. AMES

Determination of the Wedge Constant of a Stellar Photometer . . . J. A. PARKHURST

On the Types of Sun-Spot Disturbances A. L. CORTIE

On a New Form of Interferometer CH. FABRY and A. PEROT

Minor Contributions and Notes: Variability in Light of Eros, EDWARD C. PICKERING; Variation in Light of Eros, EDWARD C. PICKERING; *Nova Persei*, WILLIAM H. PICKERING; *Nova Persei* February 28 to April 4: Stonyhurst College Observatory, WALTER SIDGREAVES.

Reviews: Annals of the Astrophysical Observatory of the Smithsonian Institution, S. P. LANGLEY (G. E. H.), 280.

JUNE, 1901.

Notes on the Zeeman Effect . . . N. A. KENT

On the Motion of a *Persei* in the Line of Sight . . . . . H. C. VOGEL

The Spectroscopic Binary *Mizar* H. C. VOGEL

Standard Lines in the Arc Spectrum of Iron H. KAYSER

Observations of the Brightness of *Nova Persei* GEORGE C. COMSTOCK and JOEL STEBBINS

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Reviews: Ueber die Ursache der Nordlichter, SVANTE ARRHENIUS (A. W. Augur).

JULY, 1901.

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On a Possible Function of Disruptive Approach in the Formation of Meteorites, Comets, and Nebulae. T. C. CHAMBERLIN

The Shift of the Cadmium Line of Wave-Length 4800 Due to Pressure WILLIAM B. HUFF

On the Compound Triplets in the Line Spectrum of Mercury C. RUNGE and F. PASCHEN

The Motion of *Polaris* in the line of Sight J. HARTMAN

Minor Contributions and Notes: The Publications of the Alleghany Observatory, F. L. O. WADSWORTH; Spectrum of *Nova Persei* No. 2, EDWARD C. PICKERING; *Nova Persei* No. 2, EDWARD C. PICKERING.

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OCTOBER, 1901.

Observations and Ephemerides of the Variable Star *V Cygni* . . . N. C. DUNÉR

Peculiarity of Focal Observ tions of the Planetary Nebulae and Visual Observations of *Nova Persei* with the Forty-inch Yerkes Telescope . . E. E. BARNARD

Observations of the Earlier Spectrum of *Nova Persei* . . . . . WALTER S. ADAMS

Nebulosity about *Nova Persei* G. W. RITCHEY

Notes on Variable Stars . . J. A. PARKHURST

The Wave-Number System of Rhodium C. P. SNYDER

Spectroscopic Studies. I. EDUARD HASCHEK

Minor Contributions and Notes: Observations of the Spectroscopic Binary η Pegasi, W. W. CAMPBELL; The Orbit of the Spectroscopi-Binary η Pegasi, RUSSELL TRACY CRAWFORD; Opposition of Eros (433) in 1903, EDWARD C. PICKERING; The Inferior Conjunction of Mercury on November 4, 1901, F. RISTENPART; Publication of a New Catalogue of Variable Stars, MESSRS. DUNÉR, HARTWIG, MÜLLER, OUDEMANS; A Correction, H. SEELIGER; Concerning the Paper "On the Absorption Spectrum of Chlorine," ELIZABETH R. LAIRD.

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LXIX. November, 1901

TABLE OF CONTENTS

The Distribution of Energy in the Spectrum of the Acetylene Flame. GEORGE WALTER STEWART.

Temporary Set, as Associated with Magnetostriction. CARL BARUS.

A Preliminary Communication on the Pressure of Heat and Light Radiation. E. F. NICHOLS and G. F. HULL.

LXX. December, 1901

TABLE OF CONTENTS

Discharge of Electricity from Glowing Platinum and the Velocity of the Ions E. RUTHERFORD.

A Spectrophotometric Comparison of the Relative Intensity of Light from Carbon at Different Temperatures. ERNEST BLAKER.

Note on Electrometers. WILLIAM DUANE.

A Further Note on the Visible Radiation from Carbon. E. L. NICHOLS and ERNEST BLAKER.

New Books: *Le Chatelier:* High Temperature Measurements.

Index

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